Measurement in Economics

Theory and Applications of Economic Indices

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With 44 Figures

Physica-Verlag Heidelberg

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Part I Methodology and Methods

Testing for Aggregation Bias in Efficiency Measurement[1]

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1. Introduction

It is common practice to aggregate inputs prior to estimating the structure of production technology. It is of interest, therefore to have some idea of the impact of such aggregation on the resulting inferences concerning the structure of production technology. This information is of interest in its own right, and has been the subject of considerable research. However a knowledge of the effect of input aggregation on inferences concerning the structure of technology is valuable for another reason: since productive efficiency is measured relative to an estimated technology, input aggregation also influences one's inferences concerning the structure of productive efficiency.

There are two conditions under which input aggregation has no effect on efficiency measurement. If the price proportionality conditions of the Hicks aggregation theorem are satisfied, (see Diewert (1980)), then input aggregation has no effect on conventional measures of efficiency. Alternatively, as Färe and Lovell (1987) show, if production technology is homothetically separable, then again input aggregation has no effect on efficiency measurement. The purpose of this paper is to examine the Färe-Lovell result empirically.

The Färe-Lovell result is a straightforward extension of a result of Blackorby, Primont and Russell (1978). They were not concerned with efficiency measurement, but rather with alternative characterizations of a separable technology. They established an equivalence among a (well-behaved) homothetically separable production function, a separable cost function, and a separable distance function. While Färe and Lovell exploit separability of the distance function, here we exploit separability of the cost function. This property is of particular value for two reasons.

1Funding for this research was provided by a grant from the U.S. National Science Foundation. The authors would like to thank Jere Behrman and two readers for their helpful comments.

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ed. by W. Eichhorn

First, many empirical analyses of production estimate a cost function or a cost-minimizing equation system including the cost function. Second, two types of test for separability of a cost function already exist. Nonparametric tests have been devised by Diewert and Parkan (1978, 1985) and Varian (1984), while parametric tests have been proposed by Denny and Fuss (1977).

In this paper we estimate a flexible cost-minimizing equation system, and test various separability restrictions. We then use the test results to draw conclusions about the effect of input aggregation on measures of productive efficiency. The data base we use is a 1975-1977 panel containing 210 observations on agricultural plots in six villages located in Semi-Arid Tropical India. On each plot up to four variable inputs are combined with one fixed input to produce a single output. Two of the variable inputs are hired labor and family labor, and it is these two inputs we wish to aggregate. The aggregability of these two types of labor is an issue of longstanding interest in studies of agricultural production in developing areas; for a recent example see Deolalikar and Vijverberg (1983).

The paper proceeds as follows. In Section 2 we describe production technology and relate separability of the technology to input aggregation and efficiency measurement. In Section 3 we discuss hypothesis tests in the context of a translog variable cost function. In Section 4 our data and empirical results are discussed. Section 5 concludes with implications of our findings.

2. Separability, Aggregation and Efficiency

We begin by describing the technology relative to which efficiency is to be measured, both without and with input aggregation. The sensitivity of efficiency measurement to input aggregation turns on separability of the technology, as Färe and Lovell note. However they examine the symmetric notion of separability among groups of inputs, while we investigate the nonsymmetric notion of separability of one input group from its complement. Consequently the two analyses are somewhat different, although their implications for efficiency measurement are similar.

We assume that the production unit uses inputs $x = (x_1,\ldots,x_n) \in R^n_+$, available at fixed prices $p = (p_1,\ldots,p_n) \in R^n_{++}$, to produce a single output $u \in R_+$. Production technology is characterized by a continuous, monotonic and quasi-concave production function $\phi: R^n_+ \rightarrow R_+$. An alternative primal characterization of technology is provided by the input distance function

$$D_I(u,x) = \max \{\lambda: \phi(x/\lambda) \geq u\}, \qquad [2.1]$$

where $D_I(u,x) \geq 1 \iff \phi(x) \geq u$. The subscript "I" on the distance function D and the efficiency indexes F, O and A below indicates that they are input-based functions, and serve to distinguish them from analogous output-based functions. An equivalent dual characterization of technology is provided by the cost function

$$Q(u,p) = \min \{px: D_I(u,x) \geq 1\}. \qquad [2.2]$$

The ability of the production unit to conserve on input usage is measured by the Debreu (1951)-Farrell (1957) index of technical efficiency

$$F_I(u,x) = \min \{\lambda: \phi(\lambda x) \geq u\} = D_I(u,x)^{-1}. \qquad [2.3]$$

The ability of the production unit to conserve on cost is measured by a cost efficiency index

$$O_I(u,x,p) = Q(u,p)/px. \qquad [2.4]$$

Since cost inefficiency not attributable to technical inefficiency must be due to allocative inefficiency, an index of the latter is provided by

$$A_I(u,x,p) = O_I(u,x,p)/F_I(u,x). \qquad [2.5]$$

We now partition the input vector $x \in R^n_+$ into a subvector $x^a = (x_1,\ldots,x_k) \in R^k_+$ we wish to aggregate, and a subector $x^b = (x_{k+1},\ldots,x_n) \in R^{n-k}_+$ we do not wish to aggregate. The input price vector is partitioned accordingly, so that $p = (p^a, p^b)$. We then construct a single economic quantity index $\phi^a(x^a)$ and a single economic price index $P^a(p^a)$, each homogeneous of degree +1, that satisfy $px = P^a(p^a)\phi^a(x^a) + \Sigma_{i=k+1}^n p_i x_i$. At issue is the effect this input aggregation has on the three efficiency indexes. Insight is provided by the following result.

<u>Proposition</u> (Blackorby, Primont and Russell (1978, Theorem 3.8, p. 94): If ϕ is continuous, monotonic, and quasi-concave, then the following structures are equivalent:

(i) x^a is homothetically separable from x^b in ϕ, so that

$$\phi(x) = \hat{\phi}(\phi^a(x^a), x^b),$$

where $\hat{\phi}$ is monotonic and ϕ^a is homothetic;

(ii) p^a is separable from (u,p^b) in Q, so that

$$Q(u,p) = \hat{Q}(u,Q^a(p^a),\ p^b);$$

(iii) x^a is separable from (u,x^b) in D_I, so that

$$D_I(u,x) = \hat{D}_I(u,\ D_I(x^a),\ x^b).$$

This result is useful for two reasons. First, recall that $D_I(u,x)^{-1} = F_I(u,x)$. Then parts (ii) and (iii) establish separability conditions under which the measurement of technical, cost and allocative efficiency, using the indexes $F_I(u,x)$, $O_I(u,x,p)$ and $A_I(u,x,p)$, is invariant with respect to input aggregation. Second, parts (i) and (ii) suggest two alternative methods for determining whether these conditions are satisfied in practice. All we have to do is estimate a production function, and test for the homothetic separability of x^a from x^b, or estimate a cost function, and test for the separability of p^a from (u,p^b). However it is also worth noting a useful service not provided by this result. It provides no guidance concerning either the direction or the magnitude of the errors in the three efficiency indexes that arise when the appropriate separability conditions are not satisfied. Direction and magnitude can only be determined after estimation, to which we now turn.

3. The Translog Variable Cost Function

We use a variable cost function to investigate the effect of input aggregation on efficiency measurement, and we use the translog functional form because of its flexibility. However as Blackorby, Primont and Russell (1977) have pointed out, the flexibility of the translog functional form does not extend to the separability property--the translog is "separability-inflexible." This is a potentially serious drawback for us, since separability is at the heart of the aggregation/efficiency question. Fortunately, Denny and Fuss have provided a solution to the problem. They have shown that the separability-inflexibility of the translog applies only when the translog is treated as an exact functional form, in which case a separable translog function must be either a translog function of (inflexible) Cobb-Douglas subaggregates, or a Cobb-Douglas function of translog subaggregates. Consequently a test of the separability hypothesis based on the translog treated as an exact functional form is in fact a test of an undesirably strong joint hypothesis of separability plus Cobb-Douglas at one level or the other. However when the translog is treated as a second-order approximation to some unknown functional form, Denny and Fuss have developed a separability test that does not impose unwarranted structure on the unknown functional form being approximated.

This suggests the following sequential procedure. Estimate a translog variable cost function. Then re-estimate, imposing the Denny-Fuss approximate separability constraints. The resulting test statistic leads us to a conclusion concerning the sensitivity of the three efficiency indexes to input aggregation. If the test statistic is not significant, re-estimate again, this time imposing the stronger exact separability constraints. The resulting test statistic tells us whether the insensitivity remains when Cobb-Douglas structure is imposed on the input price index. If this test statistic is not significant, re-estimate a third time, imposing a stronger set of constraints sufficient for separability. This test statistic tells us whether the insensitivity remains when the variable cost function is a Cobb-Douglas function of translog subaggregates. Finally, for purposes of comparison, perform the input aggregation and estimate the aggregate model.

We now illustrate this procedure with a translog variable cost function tailored specifically to meet the requirements of the empirical application to be discussed below. There we analyze a sample of production units using four variable inputs and one quasi-fixed input to produce a single output. We are interested in measuring the productive efficiency of these units, and we want to know whether measured efficiency is sensitive, and if so, how, to the aggregation of two of those variable inputs. A related issue which we do not investigate, the aggregability of quasi-fixed inputs, has been analyzed by Epstein (1983), although Epstein did not consider the implications for efficiency measurement. A four-equation system, consisting of a translog variable cost function and three variable input share functions, appropriate to this environment may be written as

$$\begin{aligned} \ln(px) = {} & \alpha_o + \alpha_u \ln u + \alpha_H \ln H + \Sigma_{i=1}^4 \alpha_i \ln p_i \\ & + 1/2\ \alpha_{uu} (\ln u)^2 + 1/2\ \alpha_{HH} (\ln H)^2 + 1/2\ \Sigma_{i=1}^4 \Sigma_{j=1}^4 \alpha_{ij} \ln p_i \ln p_j \\ & + \Sigma_{i=1}^4 \alpha_{iu} \ln p_i \ln u + \Sigma_{i=1}^4 \alpha_{iH} \ln p_i \ln H, \end{aligned} \qquad [3.1]$$

and

$$S_i = \alpha_i + \Sigma_{j=1}^4 \alpha_{ij} \ln p_j + \alpha_{iu} \ln u + \alpha_{iH} \ln H, \qquad i=1,2,3, \qquad [3.2]$$

where px is variable cost, $S_i = p_i x_i/px$ is the share of the i-th variable input in variable cost, H is the quantity of the quasi-fixed input, and all other variables are as previously defined. The condition for exact separability of ($\ln p_1$, $\ln p_2$) from Z = ($\ln p_3$, $\ln p_4$, ln u, ln H) in the translog variable cost function is

$$\nabla_Z\left(\frac{\partial \ln(px)/\partial \ln p_1}{\partial \ln(px)/\partial \ln p_2}\right) = \nabla_Z(S_1/S_2) = 0, \qquad [3.3]$$

which can be written as the system of four equations

$$(\alpha_2\alpha_{1k} - \alpha_1\alpha_{2k}) + (\alpha_{1k}\ \Sigma_{m=1}^4\alpha_{2m}) - \alpha_{2k}\ \Sigma_{m=1}^4\alpha_{1m})\ \ln p_m \qquad [3.4]$$

$$+ (\alpha_{1k}\ \alpha_{2u} - \alpha_{2k}\ \alpha_{1u})\ \ln u + (\alpha_{1k}\ \alpha_{2H} - \alpha_{2k}\ \alpha_{1H})\ \ln H = 0,$$

where k = 3, 4, u, H.

A sufficient condition for this exact separability condition to hold is

$$\alpha_{1k} = \alpha_{2k} = 0, \qquad \forall k = 3,\ 4,\ u,\ H, \qquad [3.5]$$

in which case [3.1] collapses to a Cobb-Douglas variable cost function with a translog labor price index. In this case the Allen-Uzawa partial elasticities of substitution σ_{ij} satisfy $\sigma_{1k} = \sigma_{2k} = 1$ for k = 3, 4. A necessary and sufficient condition for exact separability is

$$\alpha_1\ \alpha_{2k} - \alpha_2\ \alpha_{1k} = 0, \qquad \forall k = 1,2,3,4,u,H, \qquad [3.6]$$

or, alternatively,

$$\frac{\alpha_1}{\alpha_2} = \frac{\alpha_{11}}{\alpha_{21}} = \frac{\alpha_{12}}{\alpha_{22}} = \frac{\alpha_{13}}{\alpha_{23}} = \frac{\alpha_{14}}{\alpha_{24}} = \frac{\alpha_{1u}}{\alpha_{2u}} = \frac{\alpha_{1H}}{\alpha_{2H}}, \qquad [3.7]$$

in which case [3.1] collapses to a translog variable cost function with a Cobb-Douglas labor price index. In this case $\sigma_{1k} = \sigma_{2k}$ for k = 3, 4. Finally, the Denny-Fuss condition for approximate separability is

$$\alpha_1\ \alpha_{2k} - \alpha_2\ \alpha_{1k} = 0, \qquad \forall k = 3,4,u,H, \qquad [3.8]$$

or, alternatively,

$$\frac{\alpha_1}{\alpha_2} = \frac{\alpha_{13}}{\alpha_{23}} = \frac{\alpha_{14}}{\alpha_{24}} = \frac{\alpha_{1u}}{\alpha_{2u}} = \frac{\alpha_{1H}}{\alpha_{2H}} , \qquad [3.9]$$

in which case [3.1] is a second-order approximation to an arbitrary separable variable cost function.

The procedure just described is designed to test hypotheses concerning the sensitivity of estimates of cost and technical (and hence allocative) efficiency to input aggregation. The procedure can be carried out without actually calculating these efficiency measures. However in order to gauge the direction and magnitude of errors resulting from inappropriate aggregation, and to provide a check on results when aggregation is warranted, these efficiency measures must be calculated. One way of calculating these measures is to treat the cost function as a "full frontier." That is, the four-equation system [3.1] and [3.2] is estimated, after which the estimated cost function is shifted down until all residuals are nonnegative. The transformed residuals are interpreted as percentage deviations above minimum cost, from which measures of cost efficiency can be calculated using equation [2.4]. Next, these deviations are decomposed into technical and allocative components using the Zieschang (1983) modification of an algorithm proposed by Kopp and Diewert (1982). This algorithm operates by finding a shadow price vector for which allocative inefficiency disappears; consequently it reveals the direction, as well as the cost, of allocative inefficiency. These three efficiency measures are calculated for each production unit without and with the various separability constraints. This procedure provides empirical evidence concerning direction and magnitude of sensitivity of estimated efficiency measures to input aggregation.

4. An Empirical Application

Our data base is a micro panel of agricultural production in Semi-Arid Tropical India. The data have been collected at regular intervals since 1975 as part of the Village Level Studies of the ICRISAT Economics Program, and are described in detail by Binswanger and Jodha (1978). We have data on production activities on individual plots of land located in six villages during the period 1975-1977, for a total of 210 observations. Output is an index of cereals, pulses, vegetables and other products. The four variable inputs are family labor (x_1), hired labor (x_2), seeds (x_3) and other non-labor inputs (x_4). The quasi-fixed input is the value of the plot being cultivated. In addition, the quality of the soil is controlled for with an index of eight different soil types, and year and village dummy variables are also included. Consequently most non-input sources of output variation either are accounted for or, as in the case of climate, are unlikely to vary across plots in a given year. Finally, the ratio of the price of family labor to the price of hired labor varies widely over the sample. This precludes the exploitation of the Hicks aggregation theorem as a justification for aggregating the two types of labor, and necessitates the separability tests just described.

Estimation of the four equation system consisting of the cost equation [3.1] and three share equations [3.2] was carried out by iterated seemingly unrelated regressions (ITSUR). The translog system is linear in parameters, and a linear estimator such as ITSUR imposes minimal distributional assumptions on the underlying error structure, and hence on the structure of cost inefficiencies. The system is estimated five times: in unconstrained form, with the approximate separability restrictions [3.8] imposed, with the weak exact separability restrictions [3.6] imposed, with the strong exact separability conditions [3.5] imposed, and finally as a three-equation, three input system with a labor price aggregate. In all five systems symmetry and homogeneity restrictions are imposed. In the last system the labor price index is a share-weighted sum of the prices of family labor and hired labor.

Parameter estimates for the five systems are reported in Table 1. Estimates of first-order parameters are fairly stable across models, although estimates of second-order parameters show greater variability. The primary interest of Table 1 centers on the Wald statistics of the tests of the three sets of separability restrictions. The test statistic is distributed as chi-square with degrees of freedom equal to the number of restrictions being imposed. All three test statistics are highly significant. The critical 0.01 chi-square values with 4, 6 and 8 degrees of freedom are 13.3, 16.8 and 20.1, respectively.

The approximate separability restrictions are decisively rejected. The translog cost function [3.1] is not a quadratic approximation to an arbitrary separable cost function. Since this structure is rejected by the data, so too are the more restrictive structures implied by weak exact separability and strong exact separability. There is no technological (i.e., functional separability) justification for aggregating family labor and hired labor in this context.

Another way of demonstrating failure of the separability restrictions is to examine the behavior of Allen-Uzawa partial elasticities of substitution reported in Table 2. All three types of separability impose structure on input substitution possibilities that are not apparent in the unconstrained model. In particular, two of three instances of complementarity are eliminated by the separability restrictions.

The translog cost function is not a quadratic approximation to an arbitrary separable cost function. What does this imply for efficiency measurement? Since the various separability restrictions impose unwarranted structure on production technology, they distort the standard relative to which efficiency is being calculated. It follows from the Blackorby, Primont and Russell proposition that measures of cost, technical and allocative efficiency are not invariant to input aggregation. What does not follow from that proposition is any indication of the direction and magnitude of the distortion.

To quantify the effect of the separability restrictions on the three efficiency measures, we have calculated these three measures for every observation in the unconstrained model, the three separability-constrained models, and the aggregate labor model. This involves transforming each estimated cost function into a full cost frontier in order to calculate cost efficiency, and then applying the Kopp-Diewert-Zieschang decomposition algorithm to calculate technical and allocative efficiency.

Sample means of the three types of inefficiency appear in Table 3. Frequency distributions appear in Figures 1-5. All means are small, reflecting the fact that many non-input sources of output variation are controlled for in estimation, and lending considerable support to the "poor-but-efficient" hypothesis. Also, the mean values of technical and allocative inefficiency move in opposite directions as unwarranted separability restrictions are imposed and tightened. Both means change by fully 60% as separability restrictions warp estimated efficient production isoquants (recall the results of Table 2). Overall cost inefficiency is also affected, its mean being increased by 12%. The aggregate labor results are most similar to the approximate separability results because the labor price index is consistent with approximate, but not weak exact or strong exact, separability.

The only dimension of efficiency measurement that appears invariant to the imposition of separability restrictions and input aggregation is the direction of the allocative inefficiency. In all four disaggregated models the ratios of family labor, hired labor and seeds to other non-labor inputs is inefficiently small. In the aggregated model the ratios of labor and seeds to other non-labor inputs is inefficiently small.

The sensitivity of efficiency measurement to the imposition of unwarranted separability restrictions and to unjustified input aggregation is even greater than Table 3 and Figures 1-5 suggest. Table 3 reports mean values, and Figures 1-5 depict frequency distributions. Neither tracks specific observations through the five scenarios. Furthermore, variation in individual efficiency measures through the five scenarios is greater than variation in sample frequency distributions or sample means.

5. Conclusions

Inferences concerning the structure of technology are sensitive to input aggregation. Consequently so are inferences concerning the structure of efficiency relative to that technology. The results of Färe and Lovell, based on those of Blackorby, Primont and Russell (1978), make this clear. In this paper we have developed a framework for empirical implementation of these ideas, and applied the framework to Indian agricultural production. The principal general finding of the study is that calculations of cost, technical and allocative efficiency can be quite sensitive to the imposition of unwarranted functional separability restrictions and to input aggregation that is unjustified on functional separability grounds. This sensitivity is all the more serious if interest centers on the efficiency of particular observations rather than on the efficiency of the sample as a whole.

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FIGURE 1

PLOT OF INEFFICIENCY FOR UNCONSTRAINED MODEL
TOTAL ALLOCATIVE AND TECHNICAL INEFFICIENCY

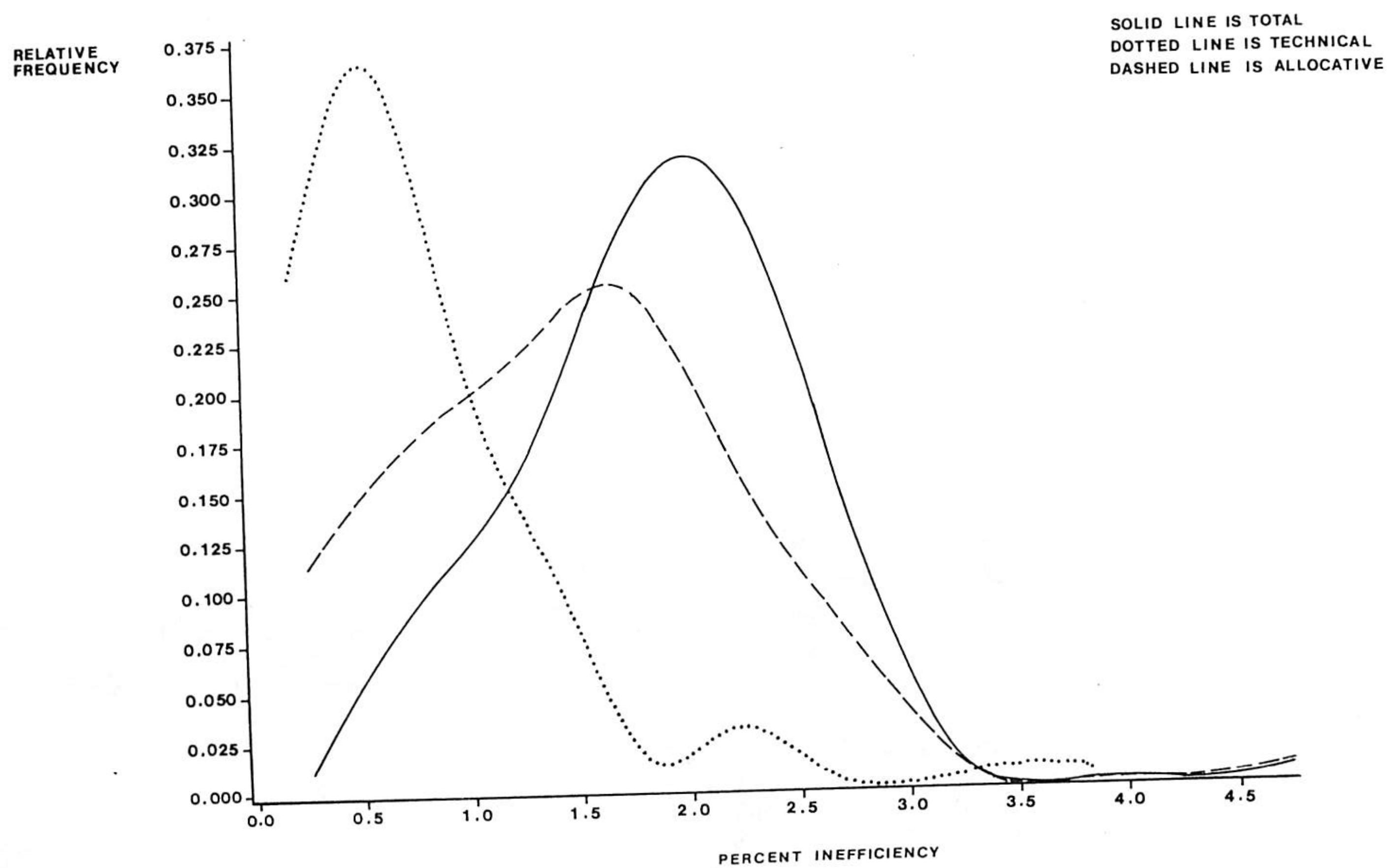

FIGURE 2

PLOT OF INEFFICIENCY FOR APPROXIMATE SEPARABILITY
TOTAL ALLOCATIVE AND TECHNICAL INEFFICIENCY

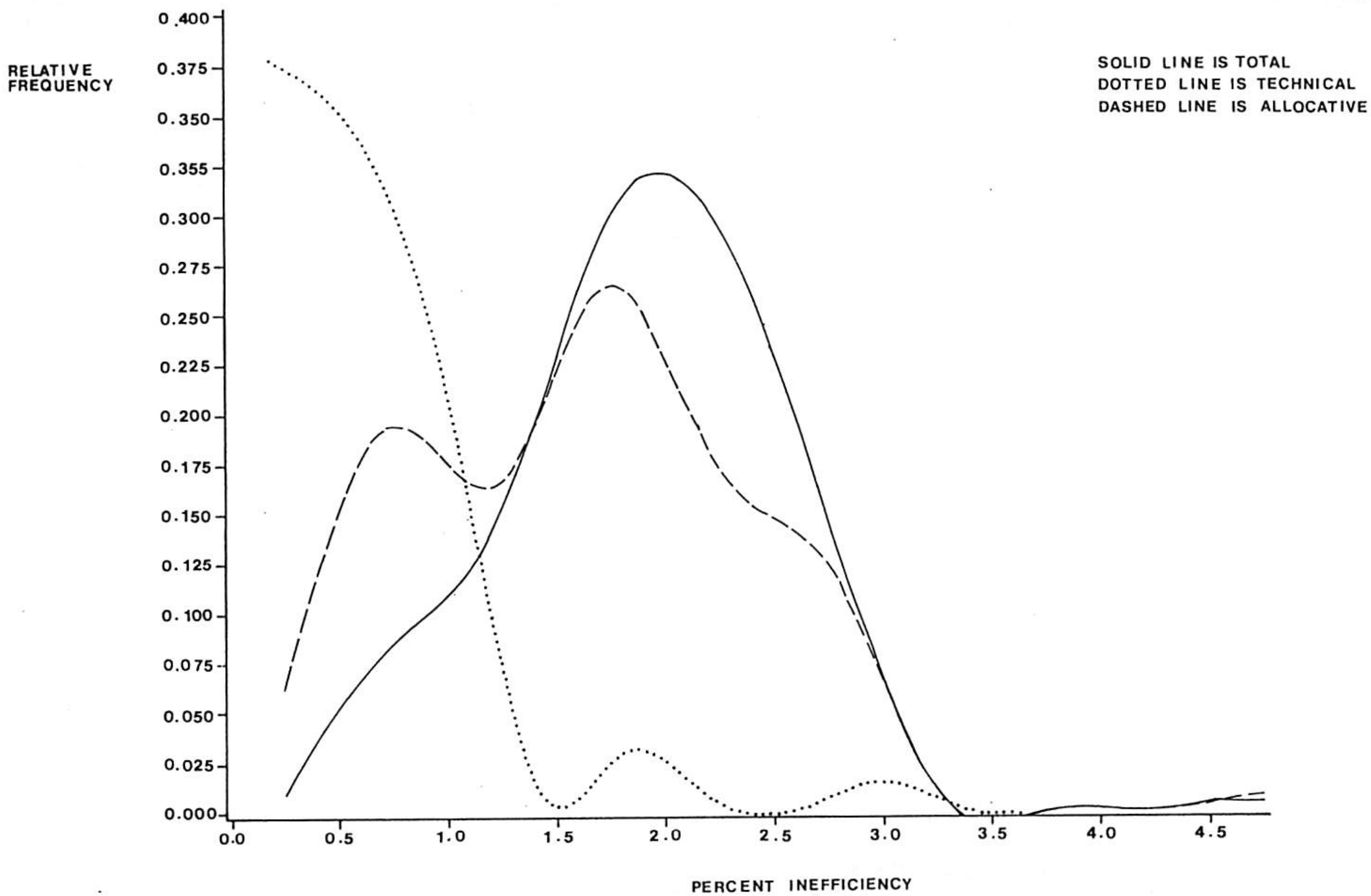

FIGURE 3

PLOT OF INEFFICIENCY FOR WEAK EXACT SEPARABILITY
TOTAL ALLOCATIVE AND TECHNICAL INEFFICIENCY

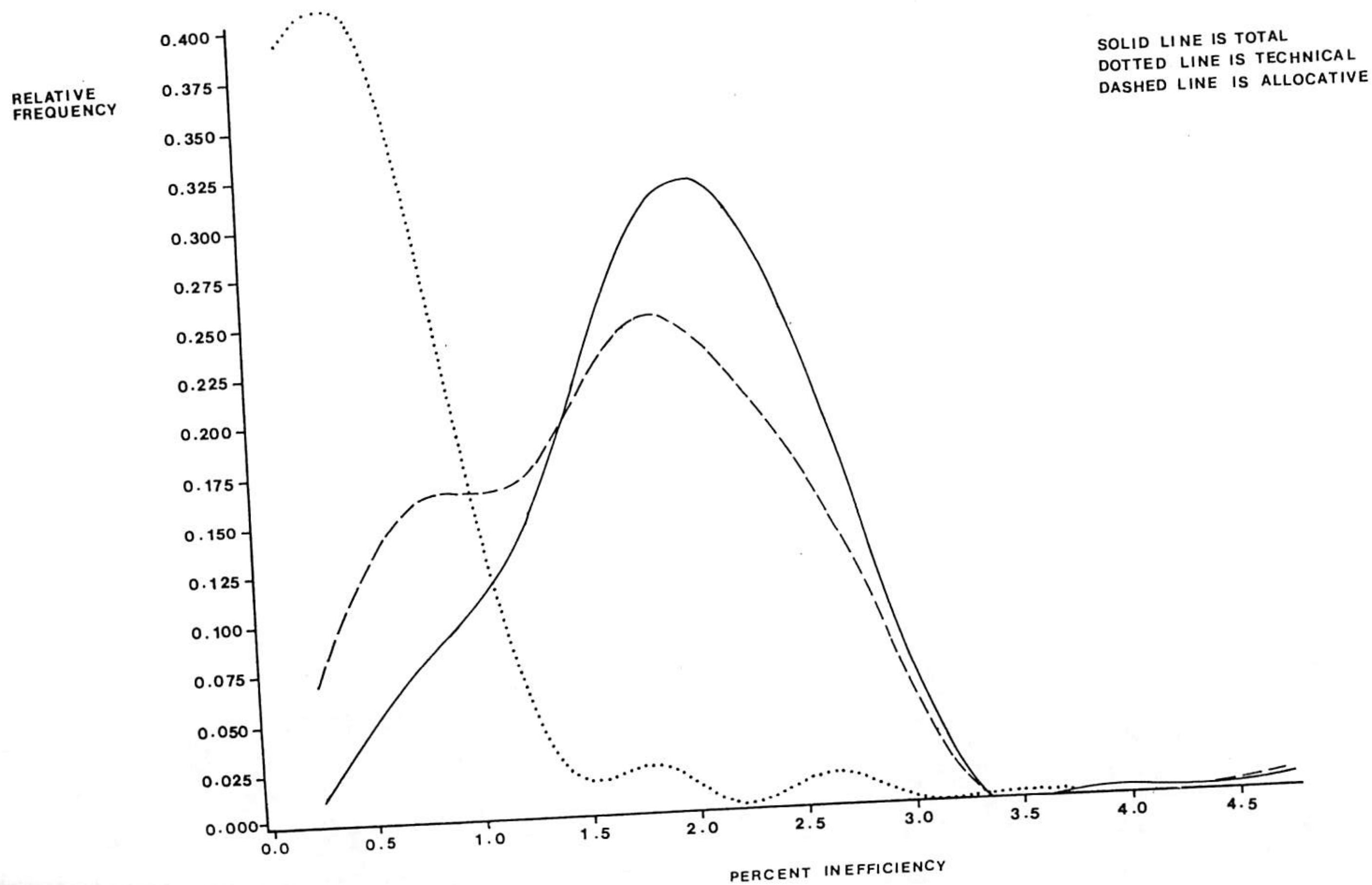

FIGURE 4

PLOT OF INEFFICIENCY FOR STRONG EXACT SEPARABILITY

FIGURE 4

PLOT OF INEFFICIENCY FOR STRONG EXACT SEPARABILITY
TOTAL ALLOCATIVE AND TECHNICAL INEFFICIENCY

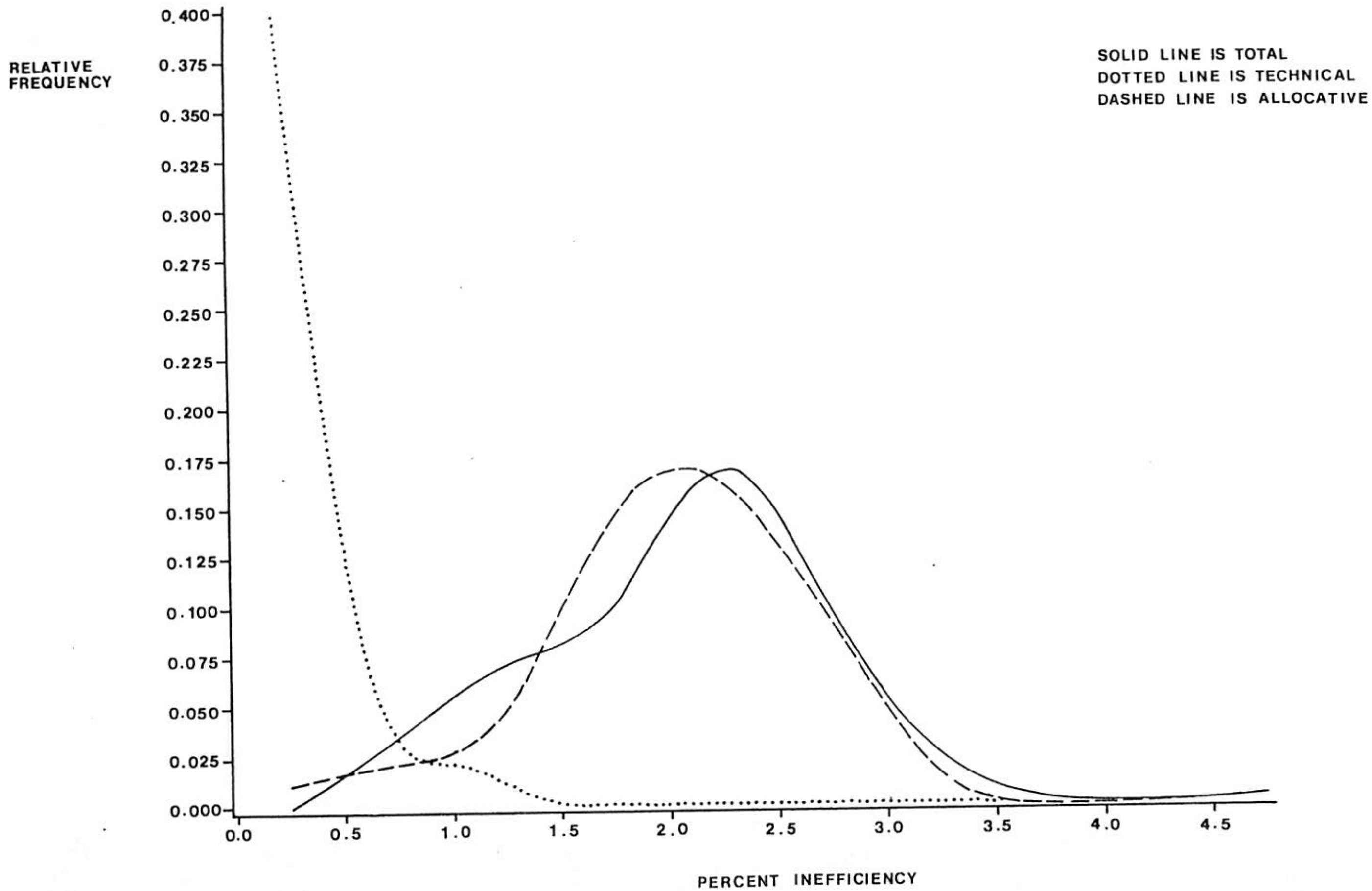

FIGURE 5

PLOT OF INEFFICIENCY FOR AGGREGRATED LABOR
TOTAL ALLOCATIVE AND TECHNICAL INEFFICIENCY

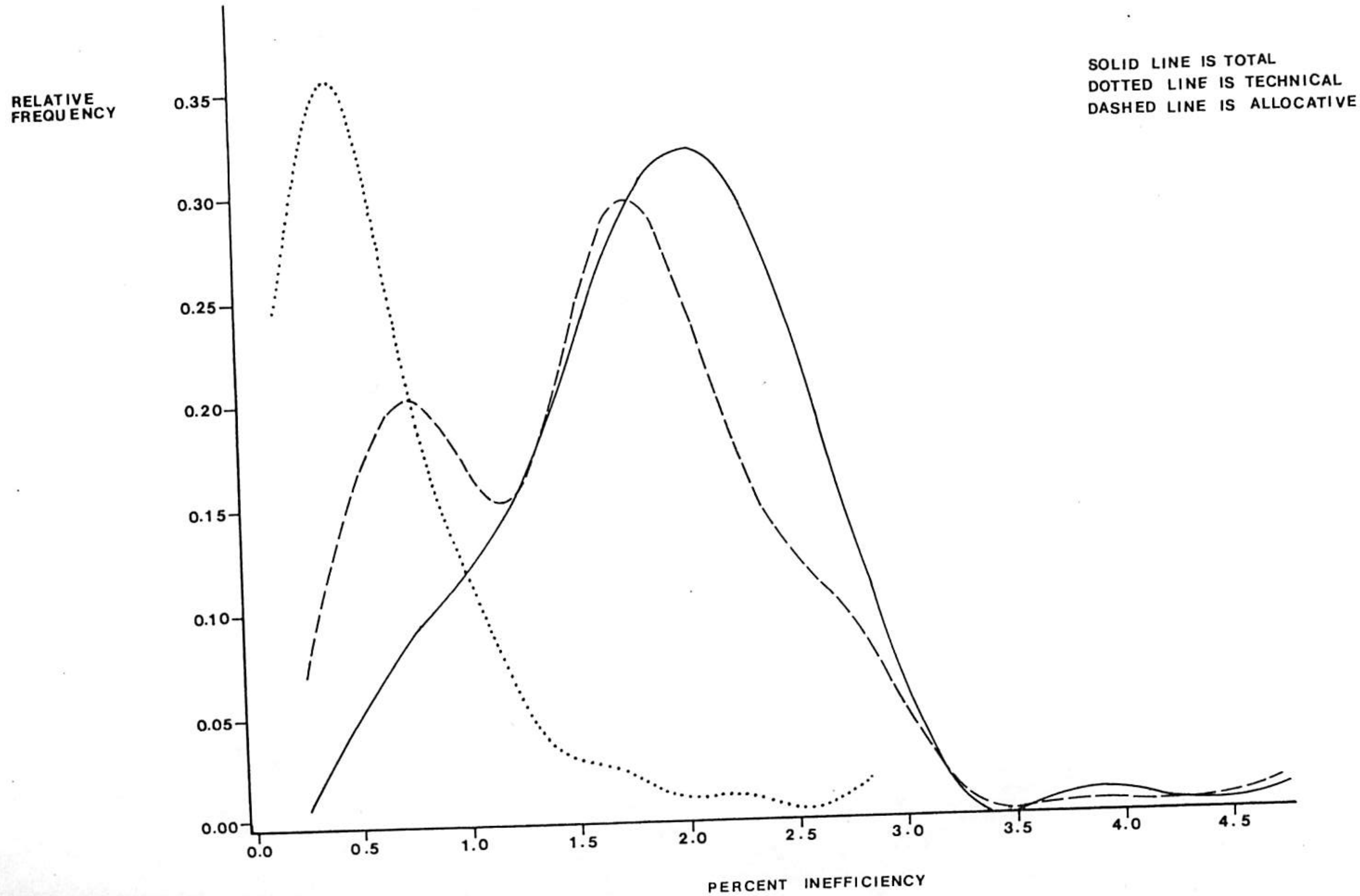

Table 1. Translog Variable Cost Functions

unconstrained approximate weak exact strong exact aggregate labor

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parameter	unconstrained model estimate	approximate separability estimate	weak exact separability estimate	strong exact separability estimate	aggregate labor estimate
α_u	0,398	0.346	0.356	0.393	0.397
	(2.35)	(1.96)	(2.01)	(2.11)	(2.32)
α_H	-1.197	-1.349	-1.261	-1.072	-0.921
	(-2.07)	(-2.24)	(-2.09)	(-1.69)	(-1.578)
α_1	0.739	0.656	0.621	0.256	1.252
	(6.02)	(9.19)	(9.32)	(16.53)	(9.71)
α_2	0.472	0.564	0.593	0.256	
	(4.18)	(8.91)	(9.35)	(19.62)	
α_3	0.390	0.381	0.386	0.542	0.388
	(3.85)	(4.04)	(4.09)	(6.34)	(3.94)
α_{uu}	0.041	0.042	0.042	0.037	0.040
	(1.50)	(1.50)	(1.46)	(1.23)	(1.46)
α_{HH}	0.168	0.188	0.177	0.152	0.132
	(2.27)	(2.45)	(2.29)	(1.87)	(1.77)
α_{11}	0.003			-0.023	0.071
	(0.05)			(5.06)	(1.67)
α_{12}	-0.035	0.095	0.016		
	(-0.77)	(2.09)	(1.59)		
α_{13}	-0.054				-0.040
	(-2,57)				(-1.93)
α_{22}	0.126	-0.070			
	(2.38)	(-1.52)			
α_{23}	0.019	-0.016	-0.016		
	(0.94)	(-1.78)	(-1.65)		
α_{33}	0.100	0.096	0.095	0.086	0.102
	(4.86)	(4.98)	(4.94)	(5.06)	(5.12)
α_{1u}	-0.078				-0.057
	(-9.68)				(-7.28)
α_{2u}	0.024	-0.027	-0.026		
	(3.21)	(-7.16)	(-6.91)		
α_{3u}	-0.015	-0.015	-0.015	-0.023	-0.016
	(-2.16)	(-2.28)	(-2.33)	(-3.71)	(-2.53)
α_{1H}	0.002				-0.029
	(0.13)				(-2.33)
α_{2H}	-0.030	-0.013	-0.014		
	(-2.62)	(-2.28)	(-2.44)		

Table 1. Translog Variable Cost Functions (continued)

parameter	unconstrained model estimate	approximate separability estimate	weak exact separability estimate	strong exact separability estimate	aggregate labor estimate
α_{3H}	-0.020	-0.020	-0.019	-0.026	-0.020
	(-1.83)	(-1.93)	(-1.92)	(-2.63)	(-1.88)
t_1	0.157	0.132	0.138	0.126	0.173
	(1.65)	(1.33)	(1.39)	(1.23)	(1.80)
t_2	0.249	0.223	0.231	0.230	0.273
	(2.63)	(2.26)	(2.33)	(2.23)	(2.84)
v_1	7.445	8.265	(7.902	7.090	6.422
	(3.22)	(3.44)	(3.28)	(2.79)	(2.75)
v_2	7.228	8.094	7.728	6.848	6.207
	(3.13)	(3.37)	(3.22)	(2.71)	(2.67)
v_3	7.305	8.204	7.838	6.911	6.276
	(3.20)	(3.46)	(3.30)	(2.76)	(2.73)
v_4	6.919	7.755	7.393	6.482	5.886
	(2.99)	(3.23)	(3.07)	(2.56)	(2.52)
v_5	7.187	7.983	7.621	6.770	6.184
	(3.10)	(3.31)	(3.16)	(2.67)	(2.65)
v_6	7.474	8.307	7.933	7.056	6.448
	(3.24)	(3.46)	(3.30)	(2.79)	(2.77)
s	0.034	0.040	0.038	0.028	0.027
	(1.05)	(1.21)	(1.13)	(0.79)	(0.84)
Wald test		58.21	137.99	175.01	

Note: t-statistics in parentheses.

Table 2. Allen-Uzawa Partial Elasticities of Substitution

Table 2. Allen-Uzawa Partial Elasticities of Substitution

	unconstrained	approximate separability	weak exact separability	strong exact separability	aggregate labor
σ_{11}	-2.917	-3.653	-2.594	-3.289	
σ_{12}	0.472	2.473	1.243	1.348	
σ_{13}	-0.055	0.653	0.685	1.0	
σ_{14}	2.212	0.859	0.791	1.0	
σ_{22}	-0.979	-4.344	-2.778	-3.226	
σ_{23}	1.359	0.669	0.685	1.0	
σ_{24}	-0.493	0.866	0.791	1.0	
σ_{33}	-1.499	-1.599	-1.620	-1.855	-1.461
σ_{34}	-0.132	-0.055	-0.086	-0.498	-0.066
σ_{44}	-1.426	-1.470	-1.328	-1.423	-1.347
σ_{LL}					-0.681
σ_{L3}					0.606
σ_{L4}					0.788

Note: σ_{ij} are evaluated at mean values of explanatory variables.

Table 3. Mean Values of Overall, Technical and Allocative Inefficiency (in %)

	unconstrained	approximate separability	weak exact separability	strong exact separability	aggregate labor
O_I	1.840	1.880	1.897	2.063	1.861
F_I	0.773	0.622	0.574	0.312	0.608
A_I	1.269	1.476	1.543	2.017	1.454

Note: O_I means based on 203 observations having positive values of output; F_I and A_I means based on 137 observations having positive values of output and all four variable inputs. Consequently $O_I \neq F_I + A_I$.