Finite sample evidence on the performance of stochastic frontiers and data envelopment analysis using panel data^{*}

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Received November 1988, final version received January 1991

In recent years a number of alternative methods have been proposed with which to measure technical efficiency. However, we know little of their comparative performance. In this study we examine the relative strengths of two different methodologies – stochastic frontier models (SF) and data envelopment analysis (DEA) – in estimating firm-specific technical efficiency. To address the limitations of previous studies we utilize Monte Carlo techniques which allow us to control the structure of the underlying technology and the stochastic environment.

Most stochastic frontier models have focused on estimating average technical efficiency across all firms. The failure to estimate firm-specific technical efficiency has been regarded as a major limitation of previous stochastic frontier models. To overcome this limitation we estimate firm-specific technical efficiency using panel data. We also examine the performance of stochastic frontier models using panel data for three estimators – maximum likelihood random effects, generalized least squares random effects, and within fixed effects.

Our results indicate that for simple underlying technologies the relative performance of the stochastic frontier models vis-a-vis DEA relies on the choice of functional forms. If the employed form is close to the given underlying technology, stochastic frontier models outperform DEA using a number of metrics. As the misspecification of the functional form becomes more serious and as the degree of correlatedness of inefficiency with regressors increases, DEA's appeal becomes more compelling. Our results also indicate that the preferred estimator for the SF model is the within estimator, which addresses two problems common to stochastic frontier models – the possible correlatedness of input levels and technical efficiency and the dependence of stochastic frontier models on distributional assumptions concerning the form of technical inefficiency.

*An earlier version of this paper was given at the April 1987 TIMS/ORSA Meetings, New Orleans. Sickles' funding was provided in part by National Science Foundation Grant SES 84-09047. The authors would like to thank William W. Cooper, C.A. Knox Lovell, Peter Schmidt, and an anonymous Associate Editor for comments that substantially strengthened the paper. The usual caveat applies.

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1. Introduction

Beginning with the work of Farrell (1957),¹ research on the measurement of productive efficiency primarily has dealt with two basic questions: how should the frontier production function of a firm or an industry be specified and how should efficiency be measured? The two issues are closely related since the production frontier is used as a yardstick for efficiency measurement.

Aigner, Lovell, and Schmidt (1977) and Meeusen and van den Broeck (1977) introduced a satisfactory conceptual basis for addressing the first question – allow for random shocks in the traditional production frontier and we utilize this specification as our reference point to measure efficiency. Theoretical guidance to the measurement of productive efficiency can be found in Debreu (1951), Farrell (1957), and Farc, Grosskopf, and Lovell (1985). Efforts to measure efficiency can be divided into two broad approaches – statistical [Schmidt (1985)] and nonstatistical [Charnes and Cooper (1985)].

We us a particular nonparametric approach referred to as Data Envelopment Analysis (DEA) which has been used widely in management science and operations research to represent the nonstatistical approach. Unlike classical statistical approaches, DEA uses linear programming techniques to envelope observed input-output data as tightly as possible without requiring *a priori* specification of functional forms. DEA requires an assumption of convexity and monotonicity of the production possibility set and employs a postulated minimum extrapolation from observed data.

DEA may provide a promising alternative technique to the usual statistical methods for estimating firm-specific productive efficiency. However, the relative superiority of specific methodologies is not just a theoretical but an empirical issue. Several papers compare results from the application of different measurement methods for the same data set [Banker, Conrad, and Strauss (1985), Banker, Charnes, Cooper, and Maindiratta (1986), and Nelson and Waldman (1986)].² The common findings of these studies are that efficiency measurement depends on the choice of functional forms to approximate the underlying technology and that it depends on the measurement methodologies employed. Unfortunately, further anlayses of these findings are hindered by the lack of knowledge of the true structure of production and efficiency. To address this limitation we utilize Monte Carlo techniques which allow us to control the underlying technology and level of firm efficiency.

¹The Farrell efficiency measure is developed by constructing a reference production set and radially measuring productive efficiency relative to it [see, e.g., Kopp (1981), Kopp and Diewert (1982), and Charnes and Cooper (1985)].

²For a recent cross-sectional comparison of corrected ordinary least squares and DEA for piecewise linear technologies see Banker, Gadh, and Gorr (1989).

Although the majority of stochastic frontier (SF) studies have focused on estimating average efficiency of all firms in an industry, works by Jondrow, Lovell, Materov, and Schmidt (1982), Waldman (1984), Huang and Bagi (1984), Schmidt and Sickles (1984), Cornwell, Schmidt, and Sickles (1990), Kumbhakar (1987, 1990), and Battese and Coelli (1988) have involved firmspecific efficiency estimation. Efficiency measurement based on cross-sectional data fails to identify unconditional firm-specific efficiency and this has been regarded as a major limitation of previous stochastic frontier models. However, there are at least two other serious drawbacks. The first is that a specific form for the distribution of productive efficiency is usually assumed in order to identify average efficiency. This means that estimation of productive efficiency can be sensitive to these a priori assumptions. The second is that efficiency is often assumed to be independent of inputs. Intuitively, this may not correspond to the behavioral assumptions of firms since a firm deciding on its choice of inputs may condition that decision on an information set which includes the perceived distribution of efficiency within the industry.

Our study focuses on the relative robustness of estimated firm-specific technical efficiency estimates using three econometric techniques – maximum likelihood estimation (mle), generalized least squares (gls), and the within estimator – and using programming techniques – DEA. We extend the comparison of the two methodologies to plausible cases in which: (1) the complexity and structures of an underlying technology differ, (2) the relative size of technical inefficiency to statistical noise in the stochastic components differs, (3) the forms of the true structure of technical inefficiency vary, and (4) input levels and technical inefficiency are allowed to have an arbitrary degree of correlation.

The remainder of the paper is organized as follows. Section 2 presents the functional forms for the underlying technology and distributional forms for stochastic components. Since applied researchers rarely have *a priori* information on the functional form of an underlying technology we use approximating functional forms, the CES-translog (CES-TL), the translog (TL), and the generalized Leontief (GL), to estimate firm-specific efficiency. In section 3 we outline three estimators and DEA as well as explain how firm-level efficiency is measured in terms of lost output or excess cost. Section 4 explains how the experiments were designed. Experimental results and their implications are discussed in section 5. Concluding remarks are given in section 6.

2. Model

To fix ideas about productive efficiency, we regard a firm to be a cost-minimizer. If a firm achieves its goal in a production activity, it is called an economically efficient firm; if a firm cannot attain its objective, it is called an economically inefficient firm. Following Farrell, a firm may fail to minimize the cost of producing its output in two ways: (1) it may be technically inefficient, failing to operate on the production frontier or cost frontier, or (2) it may be allocatively inefficient, failing to employ the least cost mix of inputs given the fixed relative input prices. In order to facilitate the comparison between SF and DEA we focus only on the former source of inefficiency and thus assume that the firm utilizes the correct input mix but that it could proportionately shrink its inputs and still produce the same level of output, i.e., the firm is technically inefficient [Schmidt and Lovell (1979)].

Under the assumption that all firms in an industry have the same deterministic production process, the representative firm's stochastic production frontier can be written as

$$y_{it} = f(X_{it}, \beta) + \omega_{it}, \tag{1}$$

where $\omega_{it} = v_{it} - u_i$, $u_i \ge 0$, $i = 1, \dots, N$, $t = 1, \dots, T$. Here y_{it} is the level of output for firm i at time t, X_{it} is the corresponding vector of inputs, β is a vector of unknown production parameters, and ω_{ii} is a random disturbance which includes statistical noise and technical inefficiency. The model is based on panel data in which a cross-section of firms (or plants) are each observed for a number of time periods. The advantage of the stochastic panel frontier model is placed on the estimation of the unconditional mean value of u_i , $E(u_i)$. The residual, $v_i - u_i$, of the stochastic cross-section (or time-series) frontier model contains noise, v_i , and cannot be used as an unconditional measure of u_i . The conditional measure, $E(u_i|v_i - u_i)$, suggested by Jondrow et al. (1982) and Waldman (1984) is dependent on distributional assumptions and still is contaminated by its dependence on the presence of stochastic technology shocks. Schmidt and Sickles (1984) showed conditions under which unconditional estimates of firm-specific technical efficiency could be consistently estimated when panel data was available and inefficiency was time invariant.

In order to estimate (1), we first need to specify $f(X_{it},\beta)$. The chosen functional form largely depends on *a priori* information about the underlying technology. Without specific engineering blue prints, etc., the choice of functional form is usually based on its flexibility [Diewert (1974) and Gallant (1981)]. The underlying technology we consider is a CRESH (Constant Ratio of Elasticity of Substitution, Homothetic) technology whose production function was introduced by Hanoch (1971) and has been used in different connections by Guilkey and Lovell (1980), Guilkey, Lovell, and Sickles (1983), and Gong and Sickles (1989), among others.

Let us suppose that a firm utilizes *m* inputs $X_{it} = (x(1)_{it}, \dots, x(m)_{it}) \ge 0$ to produce a single output $y_{it} \ge 0$, $\forall i, t$, with the technology represented by the production function

$$y_{it} e^{\theta y} it = \left(\sum_{k=1}^{m} \delta_k x(k)_{it}^{-\rho_k}\right)^{-\gamma/\rho}, \qquad (2)$$

where $\theta \ge 0$, $\gamma > 0$, $\delta_k > 0$, for all k, $\sum_{k=1}^m \delta_k = 1$. The advantage of (2) is that it includes many well-known deterministic production functions under certain parametric restrictions. For example, if $\theta = 0$ then (2) is almost homogeneous CRES (Constant Ratio of Elasticity of Substitution), if $\rho_1 = \cdots = \rho_m$ then (2) is homothetic CES (Constant Elasticity of Substitution), if $\theta = 0$ and $\rho_1 = \cdots = \rho_m$ then (2) is homogeneous CES, and it is homothetic Cobb-Douglas as $(\rho_1 = \cdots = \rho_m) \rightarrow 0$. The production characteristics of the underlying technology are summarized by returns to scale, $\gamma(x)$, and Allen-Uzawa³ partial elasticities of substitution between inputs (AES), $\sigma_{ij}(x)$,

$$\gamma(x) = \frac{1}{1+\theta y} \cdot \frac{\sum_{k} \rho_k \delta_k x(k)^{-\rho_k}}{\sum_{k} \rho \delta_k x(k)^{-\rho_k}},$$
(3)

$$\sigma_{ij}(x) = \frac{1}{(1+\rho_i)(1+\rho_j)} \cdot \frac{\sum_k \rho_k \delta_k x(k)^{-\rho_k}}{\sum_k \frac{\rho_k \delta_\kappa}{(1+\rho_k)} \cdot x(k)^{-\rho_k}}.$$
(4)

We next turn to the stochastic portion of (1). As has been noted by Aigner et al. (1977), the first of two disturbances, v_{it} , is typically assumed to be normally distributed and represents the usual statistical noise such as luck, climate, topography, and machine performance. The second, u_i , is assumed to be a nonnegative disturbance which reflects technical inefficiency. Since we have no *a priori* knowledge that the distribution of technical inefficiency has a specific nonnegative form, we examine different nonnegative distributions and analyze the robustness of our estimates of productive efficiency as its distribution varies. We follow Aigner et al. (1977), Cowing et al. (1982), and Meeusen and van den Broeck (1977) and assume that technical inefficiency is distributed with a half-normal, gamma, and exponential distribution. In addition, we also assume that technical inefficiency is time-invariant. One justification is that firm-specific inefficiency can be regarded as an inherent or structural residual between observed data and the corresponding production

³Blackorby and Russell (1989) have recently discussed the shortcomings of the Allen-Uzawa elasticities vis-a-vis Morishima elasticities. However, the Allen-Uzawa elasticities of substitution continue to be the most widely used measure of substitution possibilities.

(or cost) frontier. Without violent changes in economic environments (i.e., deregulation), firm-specific efficiency and its relative ranking are not likely to change drastically over short time periods [Seale (1985)]. Another is that the comparison between the statistical and nonstatistical methodologies is substantially facilitated by this invariance assumption. Sickles, Good, and Johnson (1986), Cornwell, Schmidt, and Sickles (1990), and Kumbhakar (1990) have recently provided estimators that allow this to be a testable hypothesis.

Since the expected value of the one-sided disturbance is nonzero, a reparameterization is necessary to insure that the composed error has zero mean. The corrected model is

$$y_{it} = (-\mu) + f(X_{it}, \beta) + (\mu - u_i) + v_{it},$$
(5)

where μ is the mean value of u_i and

$$y_{it} = (-\mu) + f(X_{it}, \beta) - u_i^* + v_{it},$$
(6)

where $u_i^* = (u_i - \mu)$. Thus the error terms v_{ii} and u_i^* have zero mean.

It is well-known that under certain conditions [Diewert (1974)] either a cost frontier or a production frontier uniquely defines the true production technology. Thus we also may estimate firm-specific efficiency using a stochastic frontier cost function which is often preferable on the basis of data availability and computational convenience. Recall that allocative efficiency is a maintained hypothesis in our analysis. Thus when using cost frontiers instead of production frontiers, the same arguments can be employed as above, except that the two error components change sign and are divided by the returns to scale. Since the cost function is by far the most widely estimated technological relationship, it will be used in our experiments. Throughout our study, we take the point of view shared by empirical modelers that we have incomplete information about the underlying technology and estimate technical efficiency using approximate functional forms. These are defined as second-order differential approximations to an arbitrary twice continuously differentiable cost function that satisfies linear homogeneity and symmetry in input prices at any point in an admissible region.

We consider three competing approximations for the cost function, the translog (TL) [Christensen, Jorgenson, and Lau (1971)], the CES-translog (CES-TL) [Pollak, Sickles, and Wales (1984)], and the generalized Leontief (GL) [Diewert (1971)]. We write the *m*-input translog (TL) as

$$\ln c(y,w) = \alpha_0 + \alpha_y \ln y + \frac{1}{2}\alpha_{yy}(\ln y)^2 + \sum_i \alpha_i \ln w_i$$
$$+ \frac{1}{2}\sum_i \sum_j \alpha_{ij} \ln w_i \ln w_j + \sum_i \alpha_{yi} \ln y \ln w_i.$$
(7)

The CES-translog (CES-TL), combines the CES and TL [Pollak, Sickles, and Wales (1984)] and thus is a hybrid form. The advantage of the CES-TL is that it is compatible with a wider range of substitution possibilities than either the CES or the TL with only one additional parameter than the conventional TL. Moreover, the CES-TL appears to have better regional curvature properties than the TL [Pollak, Sickles, and Wales (1984), Gong and Sickles (1989)]. The CES-TL is given by

$$\ln c(y,w) = \alpha_0 + \alpha_y \ln y + \ln \left[\sum_j \alpha_j w_j^{1-\sigma}\right]^{1/(1-\sigma)} + \frac{1}{2} \sum_i \sum_j \beta_{ij} \ln w_i \ln w_j + \sum_j \delta_{yj} \ln y \ln w_j.$$
(8)

The CES-TL input demand system and cost function reduce to those of the CES, if $\beta_{ij} = \delta_{yi} = 0$, $\forall i, j$. When $\sigma = 1$, the CES-TL input demand system reduces to that of the TL input demand system, and as σ approaches one, its cost function approaches the TL's. The final approximation we consider is the generalized-Leontief (GL), which we write as

$$c(y,w) = y \sum_{i} \sum_{j} \alpha_{ij} w_{i}^{1/2} w_{j}^{1/2} + y^{2} \sum_{i} \alpha_{i} w_{i}^{i} + \sum_{i} \beta_{i} w_{i}.$$
 (9)

The GL form is nonhomothetic unless $\alpha_i = \beta_i = 0$, $\forall i$, in which case it is linearly homogeneous. Thus the GL form is incapable of distinguishing among homotheticity, homogeneity, and linear homogeneity. If $\alpha_{ij} = 0$ for all $i \neq j$, then the GL collapses to a fixed proportions form. For all functions, symmetry and linear homogeneity were imposed with the appropriate parametric restrictions.

3. Three estimators and DEA

3.1. Maximum likelihood estimator

Like mle in previous stochastic frontier models, this estimator is based on several strong assumptions. First, the regularity conditions for the density functions are assumed to hold [Norden (1972, 1973)]. Second, specific parametric distributions for u_i and v_{ii} are assumed known. We choose distributions that have enjoyed the most wide-spread use in empirical applications and specify v_{ii} as normal and u_i as half-normal. Third, our likelihood derivation is based on the assumption that technical inefficiency is independent of inputs. Pitt and Lee (1981) first derived the likelihood function based on these assumptions. The density of the composed error $\omega_{ii} = v_{ii} - u_i$ is

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given by

$$\varphi(\omega_{i1},\ldots,\omega_{iT}) = \int_0^\infty h(u_i) \prod_{t=1}^T g(\omega_{it}+u_i) \,\mathrm{d}u_i, \qquad (10)$$

where

$$h(u_i) = \left(2\pi\sigma_u^2\right)^{-1/2} \exp\left(-u_i/2\sigma_u^2\right), \qquad u_i \ge 0,$$
(11)

and where $g(v_{it})$ is the normal density function with mean zero and variance σ_v^2 . Assuming independence across firms, the likelihood function is

$$L = \prod_{i=1}^{N} \varphi [y_{i1} - f(x_{i1}, \beta), \dots, y_{iT} - f(x_{iT}, \beta)].$$
(12)

Estimates of individual firm intercepts can be recovered from the estimated residuals by estimating their mean over time [Schmidt and Sickles (1984)],

$$\hat{u}_i = \frac{1}{T} \sum_{t} \hat{\omega}_{it}, \qquad i = 1, \dots, N,$$
(13)

where $\hat{\omega}_{it} = y_{it} - f(X_{it}, \hat{\beta}_{mle})$. The \hat{u}_i serve as estimates of technical efficiency. If $f(\cdot)$ contains a constant term (β_0) , then we can use the fact that $u_i \ge 0$ to normalize the effects (u_i) and the constant term (β_0) . We define $\hat{\beta}_0 = \max(\hat{u}_i)$ and define technical efficiency as $\hat{u}_i = \hat{\beta}_0 - \hat{u}_i$, i = 1, ..., N. Thus the most efficient firm in the industry is counted as 100% efficient. If the density of u is nonzero in some neighborhood $(0, \kappa)$, $\kappa > 0$, then the efficiency of the most efficient firm will approach 100% as N become large [Schmidt and Sickles (1984), Greene (1980)]. Consistency of $\hat{\beta}_0$ and \hat{u}_i requires that $N \to \infty$ and $T \to \infty$.

3.2. Generalized least squares estimator

The estimation of technical efficiency using generalized least squares is based on two assumptions: (1) technical inefficiency is regarded as a nonnegative random effect and (2) technical inefficiency is uncorrelated with the inputs. We need not specify a parametric form for the density of the random inefficiency effects as with mle.

Consider the variance-covariance matrix of the two component error vector $\omega = \text{VEC}(\omega_1, \dots, \omega_N)$, where ω_i is a $T \times 1$ vector of random ineffi-

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ciency effects for the *i*th firm:

$$\mathbf{E}(\boldsymbol{\omega}\boldsymbol{\omega}') = \sigma_{u}^{2}(I_{N}\otimes ll^{T}) + \sigma_{v}^{2}I_{NT} = \Omega, \qquad (14)$$

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where l is a $T \times 1$ vector of 1's. We use the Fuller and Battese (1973) transformation matrix,

$$P = I_T - \left(1 - \frac{\sigma_r}{\sigma_1}\right) \cdot \frac{ll^T}{T},$$
(15)

where $PP' = \sigma_v^2 \Omega^{-1}$, $P_i P'_i = \sigma_v^2 \Omega_i^{-1}$, and $\sigma_1 = T \cdot \sigma_u^2 + \sigma_v^2$. By applying the transformation *P* for all $N \times T$ observations of (6), the model becomes

$$(y_{it} - \gamma \bar{y}_i) = (1 - \gamma)\mu + \left[f(X_{it}, \beta) - \gamma \bar{f}(X_{it}, \beta)\right] + v_{it}, \qquad (16)$$

where

$$\bar{y}_i = \frac{1}{T} \sum_{t} y_{it},$$

$$\bar{f}(X_{it}, \beta) = \frac{1}{T} \sum_{t} f(X_{it}, \beta),$$

$$\gamma = 1 - \frac{\sigma_v}{\sigma_1} \quad \text{and} \quad \sigma_1^2 = T \cdot \sigma_u^2 + \sigma_v^2.$$

Nonlinear least squares (nls) is then applied to (16). Construction of firmspecific technical efficiency estimates is the same as with mle. As with mle, consistent estimation of technical inefficiency requires large N and T.

3.3. The within estimator

Because we need not assume uncorrrelatedness of inputs and technical inefficiency, the within estimator may be more appealing than its statistical competitors. From (1), we regard $(-u_i)$ as a fixed effect which is specific to the *i*th firm. The incidental parameters problem can be circumvented by using the within transformation [Arora (1973)], i.e., apply nls after expressing the model in terms of deviations from individual firm means over time. The transformed model becomes:

$$\mathbf{y}_{it} - \bar{\mathbf{y}}_i = \left[f(X_{it}, \boldsymbol{\beta}) - \bar{f}(X_{it}, \boldsymbol{\beta}) \right] + v_{it} - \bar{v}_i, \tag{17}$$

where

$$\bar{y}_i = \frac{1}{T} \sum_{t} y_{it},$$
$$\bar{f}(X_{it}, \beta) = \frac{1}{T} \sum_{t} f(X_{it}, \beta),$$
$$\bar{v}_i = \frac{1}{T} \sum_{t} v_{it},$$

or

$$w_{it} = z(X_{it}, \beta) + v_{it}, \qquad (18)$$

where

$$w_{i\iota} = y_{i\iota} - \bar{y}_i,$$

$$z(X_{i\iota}, \beta) = f(X_{i\iota}, \beta) - \bar{f}(X_{i\iota}, \beta).$$

The within estimator is based on the application of nls to (18). Firm-specific technical efficiencies are again estimated by (13) and are consistent for large N and T.

3.4. Data envelopment analysis (DEA)

DEA is a mathematical programming approach introduced to measure the productive efficiency of decision-making units (DMU's), or for our purposes firms. Charnes, Cooper, and Rhodes (1978, 1981) proposed the following measure of efficiency based on the ratio of a single output to a single input. Consider a specific firm *i* at time *t* which is to maximize a ratio of a weighted S-vector of outputs (Y_{it}) to a weighted M-vector of inputs (X_{it}) subject to the condition that similar ratios for every firm be less than or equal to unity:

$$\underset{Q,R}{\text{Maximize}} \left(Q^T Y_{it} / R^T X_{it} \right), \tag{19}$$

subject to

$$Q^T Y_{it} / R^T X_{it} \le 1,$$

$$q_s, r_m > 0, \quad \forall s, r,$$

where the weight vectors $Q = \{q_1, \dots, q_S\}$ and $R = \{r_1, \dots, r_M\}$ are the ARGMAX of (19) whose solution can be based on the nonlinear, nonconvex,

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and non-Archimedean fractional programming problem as formulated by Charnes and Cooper (1985). This later problem can be stated as:

$$\underset{Q,R}{\text{Maximize}} \left(Q^T Y_{ii} / R^T X_{ii} \right), \tag{20}$$

subject to

$$Q^{T}Y_{it}/R^{T}X_{it} \leq 1,$$

- $(Q^{T}Y_{it})^{-1}R^{T} \leq -\varepsilon l^{T},$
 $(-R^{T}X_{it})^{-1}Q^{T} \leq -\varepsilon l^{T},$
 $X_{it}, Y_{it} > 0,$

where ε is a non-Archimedean infinitesimal. Using the Charnes-Cooper transformation of fractional programming, the primal linear programming problem (DEA) is set up to:

$$\underset{\Lambda,\lambda,s^{+},s^{-}}{\text{Minimize}} \left(\Lambda - \varepsilon l^{T} s^{+} - \varepsilon l^{T} s^{-}\right), \tag{21}$$

subject to

$$Y\lambda - s^{+} = Y_{it},$$

$$\Lambda X_{it} - X\lambda - s^{-} = 0,$$

$$\lambda, s^{+}, s^{-} \ge 0,$$

$$Y_{it} > 0, \quad X_{it} > 0, \quad \forall i, t,$$

$$\lambda l^{T} = 1.$$

The primal problem (21) minimizes the intensity (Λ) of the input under the constraint that the output vector Y_{it} is enveloped from above and the input vector X_{it} is enveloped from below. After we carry out $N \times T$ optimizations we obtain solution values for the primal problem and utilize them in the measurement of firm-specific technical efficiency and production characteristics of the underlying technology. In order to determine the level of technical inefficiency we adopt the convention used by Charnes and Cooper (1985) in their *Non-Archimedean Theorem*: a firm is technically efficient if, and only if, minimizing (or optimal) values of the primal problem satisfy $\Lambda^* = 1$, $s^{*+} = 0$, and $s^{*-} = 0$, i.e., the intensity is unity and all slacks equal zero, where an

optimal solution to (21) is denoted by $(\Lambda^*, \lambda^*, s^{*+}, s^{*-})$. Inefficient firms are projected onto their efficient frontier (or efficient facet) by means of the transformation,

$$X_{it} \to X'_{it} = \Lambda^* X_{it} - s^{*-}$$
 and $Y_{it} \to Y'_{it} = Y_{it} + s^{*+}$. (22)

The movement from X_{it} to X'_{it} is a pure radial measure of inefficiency and indicates by how much inputs can be scaled down and still be able to produce the frontier level of output. Output slackness may still be in evidence, however, in that output(s) may still be increased without increasing input use. The differences,

$$\Delta X_{it} = X_{it} - X'_{it} = (1 - \Lambda) X_{it} + s^{*-} \text{ and } \Delta Y_{it} = Y'_{it} - Y_t = s^{*+},$$
(23)

represent the estimated amounts of technical inefficiency at the point (X_{it}, Y_{it}) . For the single-output technology considered herein, a necessary condition for output slackness is that the production function is piecewise linear and only weakly monotonic. Our data-generating process is based on a smooth production function that is strictly monotonic. Thus the output slackness variables (s^{*+}) are zero and the (radial) technical inefficiency measure is completely characterized by a nonunitary intensity vector (A). We construct the index of technical inefficiency for a specific firm i as the mean value of inefficiency over time. The DEA estimate of technical inefficiency is then $\hat{u}_i = T^{-1} \sum_{i} [f(\Delta X_{ii})]$, where $f(\cdot)$ is the true production function characterized in (1). Since DEA is a nonstatistical method, standard asymptotic arguments that apply to the consistency of technical inefficiency estimates using the three statistical methods above do not apply here, although in principle a weak law of large numbers argument should be applicable to prove weak convergence of the DEA estimate of technical inefficiency for large T.

4. Design of experiments

In our experiments we use the CRESH production function (2) to model the underlying true technology. We consider a technology that maps three inputs into a single output. Elsewhere [Gong and Sickles (1989)] we have examined in depth the performance of stochastic panel frontiers across a wider range of technologies than those considered in this paper. Here we limit the range of technologies to one that exhibits constant return to scale and for which the off-diagonals of the AES matrix are the same, with input substitution ranging between 3.03 and 0.333. We do this because direct comparisons between DEA and SF are somewhat clouded by the treatment of scale economies and diseconomies and because our findings for the class of technologies considered here are quite similar to results based on an

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		Functional form	
True technology	CES-TL		GL
$1. \rho = -0.67 (\sigma_{12} = \sigma_{13} = \sigma_{23} = 3.03)$	··		
Within	0.967 (0.011) ^b	0.973 (0.007)	0.726 (0.068)
GLS	0.967 (0.011)	0.976 (0.010)	0.725 (0.068)
MLE	0.980 (0.006)	0.971 (0.012)	0.726 (0.068)
2. $\rho = -0.5 (\sigma_{12} = \sigma_{13} = \sigma_{23} = 2.00)$			
Within	0.981 (0.006)	0.977 (0.006)	0.631 (0.085)
GLS	0.981 (0.006)	0.973 (0.011)	0.632 (0.085)
MLE	0.985 (0.005)	0.975 (0.007)	0.533 (0.229)
3. $\rho = -0.25 (\sigma_{12} = \sigma_{13} = \sigma_{23} = 1.33)$			
Within	0.992 (0.010)	0.988 (0.003)	0.632 (0.086)
GLS	0.981 (0.006)	0.967 (0.011)	0.619 (0.091)
MLE	0.982 (0.026)	0.957 (0.113)	0.631 (0.087)
4. $\rho = +0.1 (\sigma_{12} = \sigma_{13} = \sigma_{23} = 0.90)$			
Within	0.966 (0.011)	0.991 (0.003)	0.802 (0.049)
GLS	0.966 (0.011)	0.967 (0.002)	0.801 (0.048)
MLE	0.987 (0.003)	0.951 (0.136)	0.801 (0.050)
5. $\rho = +2.0 (\sigma_{12} = \sigma_{13} = \sigma_{23} = 0.33)$			
Within	0.976 (0.007)	0.360 (0.165)	0.985 (0.005)
GLS	0.976 (0.007)	0.302 (0.102)	0.985 (0.005)
MLE	0.972 (0.005)	0.396 (0.147)	0.985 (0.005)

Comparison of stochastic frontier models based on correlation coefficient between true and estimated relative efficiency levels.^a

^aExperiments are conducted with T = 30 and N = 50.

^bValues in parentheses are sample standard deviations of the correlation coefficients.

examination of a set of technologies exhibiting more complicated substitution and/or complementarity patterns, which results are available from the authors on request. The results reported in tables 1–10 are based on 50 replications with the number of cross-sections fixed at 50 and the number of time periods varying between 10 and 50. We have fixed $\theta = 0$, $\gamma = 1$, $\delta_1 = \delta_2$ = 0.3, and $\delta_3 = 0.4$. The inputs are drawn from a lognormal distribution and fixed throughout an experiment, as with Guilkey, Lovell, and Sickles (1983). The v_{it} are i.i.d. N(0, σ_v^2). For all simulations except those reported in tables 9 and 10, the u_i are half-normal [i.e., $u_i = |\xi_i|$ where ξ_i is NID(0, σ_{ξ}^2)] for each experiment and are constant over time. In order to consider cases in which noise in the generated data may confound the measurement of technical inefficiency as well as the situation in which technical inefficiency dominates statistical noise, we allow σ_{ξ}^2 to vary ($\sigma_{\xi}^2 = 1.03, 5.15, 15.45$) while fixing the variance of statistical noise, σ_v^2 , at 0.515.⁴

⁴As $\sigma_u^2/\sigma_v^2 \to \infty$, (1) becomes a full frontier. As $\sigma_u^2/\sigma_v^2 \to 0$, (1) becomes a stochastic production function with no technical inefficiency.

	Functional form				
True technology	CES-TL	TL	GL		
1. $\rho = -0.67 (\sigma_{12} = \sigma_{13} = \sigma_{23} = 3.03)$					
Within	0.959 (0.012) ^b	0.958 (0.017)	0.726 (0.070)		
GLS	0.958 (0.012)	0.961 (0.015)	0.725 (0.070)		
MLE	0.967 (0.011)	0.943 (0.079)	0.715 (0.107)		
2. $\rho = -0.5 (\sigma_{12} = \sigma_{13} = \sigma_{23} = 2.00)$					
Within	0.973 (0.010)	0.963 (0.014)	0.655 (0.086)		
GLS	0.972 (0.010)	0.959 (0.015)	0.657 (0.086)		
MLE	0.972 (0.012)	0.960 (0.015)	0.535 (0.277)		
3. $\rho = -0.25 (\sigma_{12} = \sigma_{12} = \sigma_{22} = 1.33)$					
Within	0.986 (0.004)	0.979 (0.008)	0.653 (0.090)		
GLS	0.972 (0.010)	0.958 (0.017)	0.658 (0.094)		
MLE	0.971 (0.033)	0.933 (0.142)	0.655 (0.092)		
4. $\rho = \pm 0.1 \ (\sigma_{12} = \sigma_{12} = \sigma_{22} = 0.90)$					
Within	0.969 (0.009)	0.983 (0.008)	0.764 (0.051)		
GLS	0.969 (0.008)	0.958 (0.006)	0.763 (0.051)		
MLE	0.977 (0.010)	0.937 (0.145)	0.756 (0.056)		
5. $\rho = +2.0 (\sigma_{12} = \sigma_{13} = \sigma_{23} = 0.33)$					
Within	0.979 (0.007)	0.330 (0.539)	0.974 (0.013)		
GLS	0.979 (0.007)	0.299 (0.477)	0.970 (0.013)		
MLE	0.978 (0.007)	0.122 (0.350)	0.970 (0.013)		

Table 2 Comparison of stochastic frontier models based on rank correlation coefficient of true and estimated efficiency levels.^a

^aSee footnote a on table 1. ^bSee footnote b on table 1.

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We generate the output observations by

$$y_{it} e^{\theta y_{it}} = \left(\delta_1 x(1)_{it}^{-\rho_1} + \delta_2 x(2)_{it}^{-\rho_2} + \delta_3 x(3)_{it}^{-\rho_3}\right)^{-\gamma/\rho} + v_{it} - u_i. \quad (24)$$

Under the assumption that each firm is a profit-maximizer, normalized input prices are generated by means of

$$w(k)_{it} = e^{-\theta y_{it}} (1 + \theta y_{it})^{-1} \frac{\gamma \delta_k \rho_k}{\rho} x(k)_{it}^{-(1 + \rho_k)} \times \left(\sum_k \delta_k x(k)_{it}^{-\rho_k}\right)^{-(\gamma + \rho)/\rho}$$
(25)

true and estimated relative efficiencies. ^a			
True technology	T = 10	T = 30	T = 50
$\overline{1.\ \rho = -0.67\ (\sigma_{12} = \sigma_{13} = \sigma_{23} = 3.03)}$			
Within	0.937 (0.023) ^b	0.967 (0.011)	0.983 (0.006)
GLS	0.938 (0.022)	0.967 (0.011)	0.983 (0.006)
MLE	0.870 (0.190)	0.980 (0.006)	0.989 (0.003)
DEA	0.744 (0.120)	0.870 (0.044)	0.897 (0.043)
2. $\rho = -0.5 (\sigma_{12} = \sigma_{13} = \sigma_{23} = 1.33)$			
Within	0.960 (0.014)	0.981 (0.006)	0.990 (0.003)
GLS	0.960 (0.014)	0.981 (0.006)	0.990 (0.003)
MLE	0.928 (0.013)	0.985 (0.005)	0.991 (0.003)
DEA	0.719 (0.118)	0.843 (0.052)	0.866 (0.057)
3. $\rho = -0.25 (\sigma_{12} = \sigma_{13} = \sigma_{23} = 1.33)$			
Within	0.974 (0.008)	0.992 (0.010)	0.995 (0.002)
GLS	0.960 (0.014)	0.981 (0.006)	0.990 (0.003)
MLE	0.938 (0.149)	0.982 (0.026)	0.993 (0.001)
DEA	0.707 (0.133)	0.816 (0.059)	0.788 (0.097)
4. $\rho = +0.1 (\sigma_{12} = \sigma_{13} = \sigma_{23} = 0.90)$			
Within	0.918 (0.034)	0.966 (0.011)	0.985 (0.003)
GLS	0.918 (0.034)	0.966 (0.011)	0.985 (0.003)
MLE	0.827 (0.068)	0.987 (0.003)	0.943 (0.007)
DEA	0.755 (0.076)	0.869 (0.043)	0.802 (0.058)
5. $\rho = +2.0 (\sigma_{12} = \sigma_{13} = \sigma_{23} = 0.33)$			
Within	0.943 (0.020)	0.976 (0.007)	0.991 (0.003)
GLS	0.944 (0.022)	0.976 (0.007)	0.991 (0.003)
MLE	0.940 (0.011)	0.972 (0.005)	0.989 (0.001)
DEA	0.665 (0.110)	0.788 (0.236)	0.701 (0.179)

			Table	3					
Comparison of stochastic	frontier	models	and D	EA	based	on correl	ation	coefficient	between
	true an	d estima	ated rel	ative	e efficie	encies. ^a			

^aThe stochastic frontier results reported here are based on the CES-TL functional form. ^bSee footnote b in table 1.

Cost data is then obtained by

$$c(y_{it} e^{\theta y_{it}}, w(1)_{it}, w(2)_{it}, w(3)_{it}) = \sum_{k} w(k)_{it} x(k)_{it}^{*}, \qquad (26)$$

where $x(k)_{it}^*$, k = 1, 2, 3, are the technically inefficient levels of the inputs. They are derived by proportionately increasing the $x(k)_{it}$ until the deterministic portion of (24) has increased by the amount of the technical inefficiency u_i . Thus the difference between $x(k)_{it}$ and $x(k)_{it}^*$ represents the amount by which inputs could be shrunk and still produce average frontier output.

5. Results

In this section we first compare mle, gls, and within as we vary the (assumed) functional form of the production function, the (estimated) ap-

Table 4

True technology	T = 10	T = 30	T = 50
1 0/7(2.02)			
1. $\rho = -0.6 / (\sigma_{12} = \sigma_{13} = \sigma_{23} = 3.03)$ Within	0.920 (0.034)b	0.959 (0.012)	0 972 (0 011)
GIS	0.920(0.034) 0.920(0.036)	0.959(0.012) 0.958(0.012)	0.972(0.011)
MLF	0.920(0.000) 0.852(0.216)	0.950(0.012) 0.967(0.011)	0.972 (0.011)
DEA	0.657 (0.105)	0.814 (0.062)	0.872 (0.051)
2. $\rho = -0.5 (\sigma_{12} = \sigma_{13} = \sigma_{23} = 2.00)$			
Within	0.949 (0.023)	0.973 (0.010)	0.982 (0.007)
GLS	0.949 (0.023)	0.972 (0.010)	0.981 (0.008)
MLE	0.910 (0.014)	0.972 (0.012)	0.983 (0.007)
DEA	0.629 (0.111)	0.775 (0.880)	0.825 (0.071)
3. $\rho = -0.25 (\sigma_{12} = \sigma_{13} = \sigma_{23} = 1.33)$			
Within	0.971 (0.012)	0.986 (0.004)	0.991 (0.003)
GLS	0.949 (0.023)	0.972 (0.010)	0.981 (0.007)
MLE	0.930 (0.142)	0.971 (0.033)	0.989 (0.006)
DEA	0.600 (0.132)	0.738 (0.082)	0.748 (0.098)
4. $\rho = +0.1 (\sigma_{12} = \sigma_{13} = \sigma_{23} = 0.90)$			
Within	0.932 (0.032)	0.969 (0.009)	0.982 (0.009)
GLS	0.933 (0.032)	0.969 (0.008)	0.982 (0.010)
MLE	0.809 (0.093)	0.977 (0.010)	0.783 (0.087)
DEA	0.763 (0.073)	0.882 (0.044)	0.783 (0.087)
5. $\rho = +2.0 (\sigma_{12} = \sigma_{13} = \sigma_{23} = 0.33)$			
Within	0.951 (0.019)	0.979 (0.007)	0.987 (0.005)
GLS	0.951 (0.020)	0.979 (0.007)	0.987 (0.005)
MLE	0.933 (0.012)	0.978 (0.007)	0.976 (0.004)
DEA	0.611 (0.123)	0.734 (0.115)	0.725 (0.101)

Comparison of stochastic frontier models and DEA based on rank correlation coefficient of true and estimated efficiency levels.^a

 $^{\rm a}{\rm The}$ stochastic frontier results reported here are based on the CES-TL functional form. $^{\rm b}{\rm See}$ footnote b in table 1.

Table 5

Comparison of stochastic frontier estimators and DEA when there is correlation between input levels and technical inefficiency (correlation coefficients used to measure performance).^a

	$C = 0.0^{b}$	<i>C</i> = 0.1	C = 0.5	<i>C</i> = 1.0
Within	0.937 (0.023) ^c	0.799 (0.144)	0.578 (0.192)	0.462 (0.180)
GLS	0.938 (0.022)	0.793 (0.142)	0.548 (0.186)	0.414 (0.172)
MLE	0.870 (0.190)	0.655 (0.299)	0.433 (0.321)	0.352 (0.318)
DEA	0.744 (0.120)	0.738 (0.092)	0.763 (0.102)	0.755 (0.093)

^aResults are based on technology #1 from table 1 with T = 10. The CES-TL is used to estimate the stochastic frontier.

^bAverage correlations between the inputs and technical inefficiency that correspond to different values of C are 0.0, -0.21, -0.24, -0.37 and are based on (27).

^cSee footnote b in table 1.

Table 6

Comparison of stochastic frontier estimators and DEA when there is correlation between input levels and technical inefficiency (rank correlation coefficients used to measure performance)."

	$C = 0.0^{b}$	C = 0.1	C = 0.5	C = 1.0
Within	0.920 (0.034) ^c	0.740 (0.117)	0.514 (0.129)	0.402 (0.120)
GLS	0.920 (0.036)	0.731 (0.117)	0.482 (0.130)	0.352 (0.133)
MLE	0.852 (0.216)	0.657 (0.297)	0.410 (0.311)	0.291 (0.320)
DEA	0.657 (0.105)	0.677 (0.101)	0.705 (0.092)	0.690 (0.101)

^aResults are based on technology #1 from table 1 with T = 10. The CES-TL is used to estimate the stochastic frontier.

^bAverage correlations between the inputs and technical inefficiency that correspond to different values of C are 0.0, -0.21, -0.24, -0.37 and are based on (27).

^cSee footnote 1 in table 1.

Table 7

Effects of the stochastic structure of technical inefficiency on the performance of stochastic frontier and DEA technical inefficiency measurement (correlation coefficient used to measure performance).^a

	Half-normal	Exponential	Gamma
Within	0.937 (0.023) ^b	0.966 (0.010)	0.971 (0.011)
GLS	0.938 (0.022)	0.966 (0.010)	0.985 (0.021)
MLE	0.870 (0.190)	0.862 (0.011)	0.817 (0.255)
DEA	0.744 (0.120)	0.824 (0.070)	0.854 (0.091)

^aResults are based on technology #1 from table 1 with T = 10. The CES-TL is used to estimate the stochastic frontier.

^bSee footnote b in table 1.

Table 8

Effects of stochastic structure of technical inefficiency on the performance of stochastic frontier and DEA technical inefficiency measurement (rank correlation coefficient used to measure performance).^a

	Half-normal	Exponential	Gamma
Within	0.920 (0.034) ^b	0.929 (0.034)	0.932 (0.001)
GLS	0.920 (0.036)	0.929 (0.033)	0.966 (0.002)
MLE	0.852 (0.216)	0.832 (0.125)	0.711 (0.357)
DEA	0.657 (0.105)	0.650 (0.094)	0.732 (0.103)

^aResults are based on technology #1 from table 1 with T = 10. The CES-TL is used to estimate the stochasic frontier.

^bSee footnote b in table 1.

Table 9

Comparison of stochastic frontier and DEA technical inefficiency measurements while varying the ratio of the variance of technical inefficiency to statistical noise (correlation coefficient used to measure performance).^a

	<i>R</i> = 2	R = 10	R = 30
SF ^b	0.937 (0.023) ^c	0.973 (0.008)	0.976 (0.007)
DEA	0.744 (0.120)	0.868 (0.051)	0.905 (0.034)
SF	0.967 (0.011)	0.990 (0.003)	0.991 (0.003)
DEA	0.870 (0.044)	0.931 (0.017)	0.945 (0.014)
SF	0.983 (0.006)	0.995 (0.001)	0.995 (0.001)
DEA	0.897 (0.043)	0.944 (0.014)	0.956 (0.010)
	SF ^b DEA SF DEA SF DEA	R = 2 SF ^b 0.937 (0.023) ^c DEA 0.744 (0.120) SF 0.967 (0.011) DEA 0.870 (0.044) SF 0.983 (0.006) DEA 0.897 (0.043)	R = 2 $R = 10$ SF ^b 0.937 (0.023) ^c 0.973 (0.008)DEA0.744 (0.120)0.868 (0.051)SF0.967 (0.011)0.990 (0.003)DEA0.870 (0.044)0.931 (0.017)SF0.983 (0.006)0.995 (0.001)DEA0.897 (0.043)0.944 (0.014)

 ${}^{a}R = \sigma_{\xi}^{2}/\sigma_{i}^{2}$ where $\sigma_{\xi}^{2} = 1.03$ and $u_{i} = |\xi_{i}|$. Results are based on technology #1 from table 1 with T = 30.

^bStochastic frontier results are based on the within estimator using the CES-TL cost function. ^cSee footnote b in table 1.

Table 10

Comparison of stochastic frontier and DEA technical inefficiency measurements while varying the ratio of the variance of technical inefficiency to statistical noise (rank correlation coefficient used to measure performance).^a

Time		R=2	R = 10	R = 30
10	SF ^b	0.920 (0.034) ^c	0.968 (0.014)	0.974 (0.011)
	DEA	0.657 (0.105)	0.851 (0.052)	0.913 (0.033)
30	SF	0.959 (0.012)	0.982 (0.006)	0.985 (0.006)
	DEA	0.814 (0.062)	0.934 (0.001)	0.964 (0.014)
50	SF	0.972 (0.011)	0.990 (0.004)	0.992 (0.004)
	DEA	0.872 (0.051)	0.959 (0.018)	0.978 (0.010)

^a $R = \sigma_{\xi}^2 / \sigma_v^2$ where $\sigma_{\xi}^2 = 1.03$ and $u_i = |\xi_i|$. Results are based on technology #1 from table 1 with T = 30.

^bStochastic frontier results are based on the within estimator using the CES-TL cost function. ^cSee footnote b in table 1.

proximating form for the cost function, the number of time periods, the relative variance of technical (u_i) to stochastic (v_{ii}) error $[\sigma_{\xi}^2/\sigma_{v}^2]$, where $u_i = |\xi_i|$ and ξ_i is NID $(0, \sigma_{\xi}^2)$], the density of the underlying stochastic inefficiency (u_i) , and the correlation between the regressors and stochastic inefficiency. We next compare the performance of SF and DEA. The most technically efficient firm in the sample is normalized by SF and DEA to be 100% efficient. We summarize the performance of alternative estimators of technical inefficiency using the correlation and rank correlation coefficient between true and estimated inefficiencies. These summary statistics measure the relationships that are of central importance to this paper, i.e., the degree

to which statistical and nonstatistical efficiency estimates are in concordance with the true data-generating process. The correlations also allow us to condense a substantial amount of information concerning sets of technical inefficiency estimates of 50 firms into a manageable number of tables.^{5,6}

Tables 1 and 2 provide correlations and rank correlations between the true and estimated relative efficiency levels as we vary the true technology, the cost function's functional form, and the estimator. None of the estimating forms are self-dual to the primal functional form of the production function, but rather are approximations whose performance cannot be evaluated analytically, unless we estimated the primal directly or utilized self-dual forms such as the Cobb-Douglas or CES. We assume that the distribution of inefficiency has been properly specified in the likelihood equation and that there is no correlation between the regressors and inefficiency. We normalize the variance of technical inefficiency (σ_u^2) so that $\sigma_{\xi}^2/\sigma_v^2 = 2.0$, where $u_i = |\xi_i|$. Tables 9 and 10 consider experiments in which this ratio varies from 2.0 to 30.0.

Our results indicate that there is no particular advantage of mle or gls over the within estimator, which is attractive on *a priori* grounds in that it makes no assumption that the firm effects and regressors in the cost dual are uncorrelated and does not assume a parametric form for the mixture distribution. Our results also indicate that the performance of stochastic frontier models depends on the structure of the underlying technology, since any bias that may be introduced by employing an inappropriate functional form will distort efficiency measurement. Evidence on the fairly simple technologies reported here indicate that, except for the case close to a fixed coefficient technology, the performance of the CES-TL and the TL dominates the GL over a wide range of relatively simple technologies. As the underlying technology approaches the Leontief, however, we find that the GL does a relatively better job of estimating firm-specific technical efficiency than the CES-TL or the TL. The performance of the TL also deteriorates markedly in the case of a fixed coefficient technology. These results have some intuitive appeal. The Cobb-Douglas is the only CES form compatible with the TL form. The CES-TL provides a transparent generalization of the CES and the TL and outperforms the TL except for a special case of the underlying technology, Cobb–Douglas. The GL is better than the two other forms in the characterization of a fixed coefficients underlying technology. Thus, in stochastic frontier models the choice of the proper functional form to

⁵Estimates of technical efficiency from the stochastic frontier model could presumably be refined by utilizing distributional assumptions on u_i after estimation. This would be done using the conditional estimator considered by Jondrow et al. (1982) and Battese and Coelli (1988). Whether or not this would have much effect on the performance of the mle estimator in these experiments is unclear. The authors wish to thank Peter Schmidt for pointing this out to us.

⁶More detailed results are available from the authors on request.

characterize an underlying technology is important in deriving unbiased information about firm-specific technical inefficiency. Based on the results provided here and elsewhere [Pollak, Sickles, and Wales (1984), Gong and Sickles (1989)], the CES-TL would appear to provide better estimates of the inefficiency characteristics of technology over a wider range of the space of technologies than either the TL or the GL.

Tables 3 and 4 provide comparisons of the three SF estimators as we vary the time period over which technical inefficiency is assumed constant. In these and subsequent tables we use the CES-TL as the estimating form for the cost function. We note that all three SF estimators provide relative technical inefficiency estimates and rankings that are in strong concordance with the data-generating process across this set of relatively simple technologies as T increases, although gls and mle appear to improve relatively more as we move from samples of T = 10 to those of T = 50. The pattern of these results are quite similar when more complicated technologies are considered. We note, however, that the reliance of the consistency of relative technical inefficiency estimates on large T is a shortcoming of all three estimators since the assumption that technical inefficiency is time-invariant is more problematic as we increase the length of our panel.

We next turn to the effect that correlation between input levels and technical inefficiency may have on the relative performance of the mle, gls, and within SF estimators. Tables 5 and 6 consider a technology in which substitution possibilities are relatively easy (AES = 3.03). We assume that the dependence of inputs two and three on technical inefficiency takes the form

$$\tilde{x}(2)_{it} = (1 + C/u_i)x(2)_{it}, \qquad \tilde{x}(3)_{it} = (1 + 2C/u_i)x(3)_{it}, \qquad (27)$$

and thus the relative differences between uncorrelated and correlated inputs is an increasing function of C and inversely related to the level of technical inefficiency. The time period is fixed at T = 10 and C takes on values of 0.0, 0.1, 0.5, and 1.0, corresponding to average sample correlations between these two inputs and technical inefficiency of 0.0, -0.21, -0.24, and -0.37. As the magnitudes of the correlations increase, the performance of all SF estimators deteriorate noticeably. This is somewhat counterintuitive since the within transformation is designed to remove the correlation and so should be robust to this type of misspecification. Were we estimating the production function this reasoning would be correct. However, introducing correlation between technical inefficiency and the level of inputs means that technical inefficiency is no longer neutral and thus normalized prices based on (25) correspond to nonneutral and hence nonradial technical inefficiency. This is an important issue. Such correlations as these distort the measurement of technical efficiency based on the cost dual since prices correspond to inputs that are both allocatively and technically inefficient relative to the data-generating process that assumes no such correlation. It is also important to note that econometric approaches to estimating frontier cost functions may well loose any advantage over programming approaches such as DEA when such correlation patterns exist in the data. The correlations used in our experiments are rather small, ranging from -0.21 to -0.37, but even with such relatively benign levels of correlation DEA measures of technical efficiency are much closer to the true levels than those estimated from stochastic frontier cost equations.

An alternative experimental design could have been adopted in which efficiency was measured relative to the cost function instead of the (nonselfdual) production function. In this case one of our findings – that all of the stochastic frontier estimators performed relatively poorly when correlation between inefficiency and regressors increased – would certainly be different. In particular, the within estimator for cost function stochastic frontiers eliminates this type of misspecification while mle and gls do not, thus enhancing the appeal of within SF estimates vis-a-vis those obtained from DEA.

We next vary the structure of technical inefficiency and assess its affect on the three SF estimators. Tables 7 and 8 consider the same technology as we considered in tables 3 and 4 with T = 10. Results for panel designs with larger numbers of time series observations are in general qualitatively quite similar to those considered in these tables. In addition to the half-normally distributed technical inefficiency considered in tables 1-4, we also consider exponential and gamma distributed technical inefficiency. The distributions are parameterized so that the mean levels of technical inefficiency are the same, although the variances do differ, with the variances for the exponential and gamma somewhat larger than for the half-normal.⁷ The likelihood function is constructed under the assumption that technical inefficiency is half-normal. The within and gls estimators are effectively the same for the half-normal and exponential case while gls provides a slight advantage over within when technical efficiency is gamma distributed. When the underlying mixture is properly specified mle is no better than either within or gls, although the relative ranking of mle switches as the time series is lengthened which is consistent with asymptotic theory. Both gls and within dominate mle when the mixture density is misrepresented. We would conclude that the within estimator is preferred on *a priori* grounds and on the basis of our experimental results.

⁷Our base case assumes that ξ_i is NID(0, 1.03). Thus $u_i = |\xi_i|$ has mean 0.810 and variance 0.37. For the exponential distribution the mean is 0.810 and the variance is 0.656. We use a special case of the gamma distribution [Greene (1980), Stevenson (1980)] with shape parameter equal to unity. We set the mean of the gamma distribution at 0.810. A unitary shape parameter forces the variance to also equal 0.810.

We next consider the relative performance of SF and DEA. In tables 3 and 4 we examine the relative merits of DEA and different estimators of the SF model as we vary the number of time periods and the underlying technology. Comparisons between DEA and different SF flexible forms can also be made by focusing on results for T = 30 in tables 3 and 4 and in those from tables 1 and 2. Across the technologies considered here the CES-TL consistently outperforms DEA using our performance metrics. There are, however, a number of situations in which DEA either dominates or is comparable to the TL or GL. In particular, for all cases except when the technology becomes close to fixed coefficients (technology #5), DEA outperforms the GL stochastic frontier, and it outperforms the TL and presumably its nested special case, the Cobb–Douglas which has been used so often in stochastic frontier applications for technology #5. There is a convergence of sorts between the two methodologies as the number of time periods increases in that the relative differences between the CES-TL SF and DEA fall. However, as we have mentioned above, in applied work the assumption of time invariance in efficiency levels and rankings becomes more difficult to justify as we lengthen the time series.

Tables 9 and 10 consider the effects of changes in the ratio of the variance of technical inefficiency to statistical noise and the number of time periods using the CES-TL within estimator and using DEA. DEA assumes that the data are free from the usual types of statistical noise, such as measurement error or the effects of omitted inputs. In such cases statistical noise may be labelled as inefficiency and thus the performance of DEA may be contaminated by its presence. We can see from the tables that as the number of time periods increases, the relative performance of both methods improves markedly as we would expect since the efficiency measures are based on sample means over time. We can also see that as the variance of inefficiency increases relative to statistical noise, the gap between the performance metrics for the two methods narrows substantially. On the basis of results from tables 5 and 6 it is clear that DEA is not sensitive to variations in the degree of the correlation between inputs and technical inefficiency since DEA does not use the contaminated price data. For these relatively simple technologies the results can be taken as encouraging evidence of DEA's viability vis-a-vis SF cost models. Tables 7 and 8 reveal a similar robustness of DEA to variations in the stochastic structure of technical inefficiency, although given the distribution-free nature of DEA it may seem surprising that there is any variation in DEA's performance as we vary the distribution of technical inefficiency. However, as pointed out above, the variance of technical inefficiency for the exponential and our parameterization of the gamma is forced to vary as we maintain a constant average level of technical inefficiency (footnote 7). That DEA appears to improve somewhat in tables 7 and 8 as we move from the half-normal to the gamma and from the exponential to the gamma is most likely due to the increase in the variance of technical inefficiency between these two sets of inefficiency distributions while holding the variance of statistical noise constant. As the results of tables 9 and 10 suggest, DEA should improve vis-a-vis SF when the ratio of the variance of technical inefficiency to stochastic noise increases and this is precisely the case in tables 7 and 8, with the ratios at 0.718, 1.27, and 1.57 for the half-normal, exponential, and gamma, respectively. That a similar relative improvement in DEA does not occur when we move from the half-normal to the exponential in table 8 may be an artifact of the relative variations of rank correlation coefficients for the two sets of experiments.

6. Conclusion

This paper has analyzed how well stochastic frontier and data envelopment methodologies estimate firm-specific technical inefficiency. To overcome the limitations of the bulk of previous comparative research we have utilized Monte Carlo techniques and have extended our comparative study to several plausible cases of misspecification. We summarize the results and their economic implications as follows:

(1) The choice of functional form in stochastic frontier models appears to be crucial. The CES-TL outperforms other competing parametric forms considered here and DEA over a wide range of the underlying technologies. Except for the case close to a fixed coefficient technology, DEA outperforms the GL. For the case close to a fixed coefficient technology DEA dominates the TL.

(2) Efficiency estimates using the within estimator are very similar to the gls and mle SF estimators. We can ameliorate two common problems of previous stochastic frontier models with the within estimator. That is, we do not have to assume an arbitrary structure for the distribution of technical inefficiency in order to separate statistical noise and technical inefficiency and we need not assume that input levels and technical inefficiency are uncorrelated. However, when using the cost dual as we do in our experiments, prices that correspond to profit-maximizing levels of the technically inefficient input levels are not those that support a radial measure of technically inefficient input use. Thus the within transformation does eliminate potential correlation of prices and technical inefficiency from the cost function, but the correlation of technical inefficiency and input levels introduces nonradial allocative inefficiency which cannot be identified by any of the SF estimators using the contaminated price data.

(3) DEA does not rely on price data to construct the efficient frontier. Thus when the researcher suspects that correlations may exist between technical inefficiency and certain inputs, DEA may well be preferred over SF cost functions on *a priori* grounds. In this case the SF production function should be estimated directly using the within estimator, or a suitable instrumental variables analogue [Cornwell, Schmidt, and Sickles (1990)]. If, on the other hand, the correlation is suspected to be between input prices which support the radial measure of technical inefficiency and technical inefficiency itself, then the within estimator should be used on the SF cost function. Since the applied researcher rarely has such foreknowledge, DEA can have substantial appeal.

(4) In contrast to previous DEA studies, we assume that the panel of firms contain a certain amount of statistical noise and allow DEA's performance to depend on the ratio of technical inefficiency to statistical noise as well as the number of time periods. The presence of statistical noise plays an important role in DEA which assumes a deterministic frontier. As the time periods increases, however, the effect of statistical noise on DEA efficiency measurement is mitigated.

We hope that the results of our Monte Carlo analyses will be informative to the applied researcher interested in the choice of appropriate methods with which to estimate firm-specific inefficiency. Our results are based on what we believe to be a representative experimental design. We note however that (1) the known underlying technology is assumed to be a CRESH production function, (2) the generated data are assumed to contain statistical noise, (3) the data are assumed to be from a panel in which firm-specific technical inefficiency is constant over time, and (4) technical inefficiency is assumed to have particular nonnegative distributions. Although we feel that these are reasonable assumptions on which to base our data construction, we cannot discount the possibility that our findings would change if the experimental design were altered or if a wider range of technologies were considered. However, our experimental evidence suggests that the tracking ability of either SF or DEA deteriorates rapidly as the underlying technology becomes more complex. Thus our caveats on the shortcomings of various estimators under various forms of misspecification are most likely conservative.

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