

# Consistent Price Determination of Income and Production: A Singular Equation System Applied to the MPS Model

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In this article we present a general methodology that can be used to estimate a singular equation system of relative prices for a large disaggregated macroeconomic model (MPS). The accounts consistency requirements and the necessity to utilize distributed lag restrictions and to impose a serial correlation structure on the estimated model make these sum constraints rather cumbersome. Estimates of the MPS model and multiplier analysis illustrate the use of these constraints on a large forecasting model as well as the feasibility of the technique.

## INTRODUCTION

When constructing large equilibrium econometric models theoretical concerns are usually weighted against the practicality of implementation. One such concern is the balancing of production and expenditure accounts in each period. This is a particularly difficult set of restrictions to impose on a system in which quantities and prices are endogenous. Unfortunately these restrictions are also quite important, especially if simulations are to be performed with the model.

The ex-post requirement that total value added in production must equal the aggregate value of final expenditure has not been imposed explicitly in most large scale models. In this article we outline one approach to this problem which has been successfully implemented with the MIT-PENN-SSRC model (MPS). This approach allows us to determine production and expenditure prices in a singular equation system with feedbacks (Berndt 1975). The dynamic nature of the model forces us

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to impose a structure on the lag distributions of the sector prices which are consistent with the ex-post sector equilibrium conditions. These are not transparent restrictions since the lag distributions used are stochastic polynomials (Shiller 1973).

In Section 1 we discuss previous methods used to maintain accounting consistency between production and expenditure prices. Section 2 outlines the singular equation system of relative prices. We also discuss the properties of the relative price system in its estimable form. Section 3 presents estimates of a version of the system used within the MPS model. In Section 4 we examine the stability of the model and the implied long-term effects of various policy shocks.

## 1. CONVENTIONAL PRICE DETERMINATION AND MODEL CONSISTENCY

In disaggregated equilibrium models total value added in production must equal total expenditure on goods and services. If we let  $x_i^d$  ( $i = 1, \dots, n$ ) be the expenditure in the  $i$ th final demand expenditure sector and  $y_j^s$  ( $j = 1, \dots, m$ ) be the value added by the  $j$ th producing sector, then

$$\sum_{j=1}^m y_j^s = \sum_{i=1}^n x_i^d. \quad (1)$$

Let  $p_i^d$  be the demand price in the  $i$ th expenditure sector and  $p_j^s$  be the supply price in the  $j$ th production sector. Then we can write

$$\sum_{j=1}^m y_j^s p_j^s = \sum_{i=1}^n x_i^d p_i^d. \quad (2)$$

Typically, in most large econometric models separate real final demand equations are specified for each  $x_i^d$ . Let these be given by

$$x_i^d = D_i^d(\cdot). \quad (3)$$

Given the structural demand equations, the aggregate value added prices are determined through structural equations of the form

$$p_j^s = S_j^s(\cdot). \quad (4)$$

The task remains to determine the individual final demand sector prices  $p_i^d$  while adhering to the necessary consistency of the accounts balance in (2). One approach to this task is to relate the expenditure prices to one or several value-added production price deflators either through constant exogenous ratios or simple linkage equations. The consistency problem is then handled by adjusting either production or expenditure prices after simulation when the accounts are tallied. Examples of this approach may

be found in early versions of the Canada Trace Mark III model, the Wharton model, and the MPS model.

The shortcoming of relating all final demand expenditure prices to one driving production sector price equation is that the method ignores the influence of other value-added prices on final expenditure prices. Moreover, an extreme assumption is made in determining final demand prices. These must conform to the determinants of the driving value-added price equation up to a fixed level of proportionality. This ignores the disparate market conditions in each demand sector.

Among the alternative approaches, one is based on the determination of final demand prices through input-output relationships from all value added output prices. Examples of applications of this method are found in Kresge (1969) for the Brookings model, Sato (1969) for the Wharton model, Preston (1972) for the Wharton Annual and Industry Forecasting Model, and Berner et al. in the contexts of a multicountry trade model (1977) and a modified version of the MPS model (1975). This approach recognizes that in the construction of equation (1) a real unit of value-added output must be consumed totally by proportions of the final demand expenditure sectors

$$y_j^s = c_{1j} x_1^d + c_{2j} x_2^d + \dots + c_{nj} x_n^d, \quad (5)$$

where  $\sum_i c_{ij} = 1$  and  $c_{ij}$  represents the proportion of value added output originating in the  $j$ th production sector consumed in the  $i$ th final demand expenditure category. For the system as a whole, the form of (5) is

$$Y = CX, \quad (6)$$

where  $Y$  is a  $m \times 1$  vector of  $m$  production sector outputs,  $C$  is a  $m \times n$  matrix of the derived input-output coefficients, and  $X$  is a  $n \times 1$  vector of final demand expenditures. Let  $D$  be the dual of  $C$ . The elements of  $D(d_{ji})$  represent the proportion of expenditures in the  $i$ th final demand category attributable to the value-added output originating in the  $j$ th production sector for the  $i$ th final demand sector

$$x_i^d = d_{1i} y_1^s + d_{2i} y_2^s + \dots + d_{mi} y_m^s. \quad (7)$$

Across  $n$  final demand categories  $\sum_i d_{ji} = 1$ . Using the  $d_{ji}$  coefficients as weights, the  $i$ th final demand expenditure price deflator is expressed in terms of the  $j = 1, 2, \dots, m$  value-added production prices.

$$p_i^d = d_{1i} p_1^s + d_{2i} p_2^s + \dots + d_{mi} p_m^s. \quad (8)$$

When final demand prices are expressed as a linear combination of value-added prices, homogeneous of degree one, the accounts consistency

requirement of (2) is maintained. There are, however, several limitations to the input-output approach. Kresge (1969) notes that the derived  $d_{ji}$  coefficients are not time invariant with respect to the production-expenditure mix within the economy. Since the input-output classifications do not exactly replicate the national income account based expenditure classifications, a statistical discrepancy will remain in the balance identity. Kresge (1969), and later Berner (1976), used iterative techniques to adjust the  $d_{ji}$  coefficients in order to balance the accounts. In applying Klein's homogeneity assumption (1969), Sato notes that the assumption of time-invariant weights is quite innocuous. This assumption requires the derived fixed weight input-output coefficients to be homogeneous of degree one in prices in the long run.

It would seem intuitively appealing to begin determining final demand expenditure prices at the level of market disaggregation inherent in the accounts of the model. Although the link between value added prices and expenditure prices is straightforward, most macro models do not have production prices entering as arguments in the market behavior of the final demand expenditure sector. The input-output relationships, as will be shown below, can be useful in distributing these effects.

## 2. A SINGULAR EQUATION SYSTEM OF RELATIVE PRICES

The consistency of the production and expenditure accounts in an econometric model can be maintained by utilizing a singular equation system of relative prices. In such a system demand sector prices are estimated relative to either the production sector prices or a consistent linkage to these output prices. A singular equation system,<sup>1</sup> as defined by Berndt and Savin (1975), is one in which for each observation the sum of the regressands is equal to a linear combination of particular regressors.

We use such a singular equation system to estimate the MPS model's price sector. We first define a total fixed weight price deflator as a weighted average of final demand prices. The structural expenditure price

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<sup>1</sup>Singular equation systems are found in models of financial asset behavior which are constrained by total asset wealth within the system. Early examples of these models are found in Brainard and Tobin (1968) and later in Gramlich and Kalchbrenner (1970). Other examples are consumer demand systems, bank portfolio behavior systems constrained by budget wealth, and models in which either factor share or market share equations are specified. For large scale econometric models, singular equation systems have been used for estimating bank portfolio and international capital stock models (see, e.g., Fitzgerald 1978 and Urdang 1979).

equations are then estimated relative to the total fixed weight deflator by sector weight.

It should be noted that although demand prices are homogenous of degree one in all value added prices, some demand sectors may be more closely related to specific value added sectors than are others. Final demand prices in the consumption expenditure sector, for example, would be more highly sensitive to market conditions operating in manufacturing than those that determine farm production prices. Since we wish to estimate the sensitivity of market clearing prices within each final demand category to the determinants of value-added prices, which in turn relate to the individual expenditures markets, we restate (2) as

$$y_1^s p_1^s = \sum_{i=1}^n x_i^d p_i^d - \sum_{j=2}^m y_j^s p_j^s, \quad (9)$$

where, without loss of generality,  $p_1^s$  can be considered as the value-added price of the production sector whose cost function most clearly parallels the market pricing in the final demand expenditure sectors. The fixed weight deflator is then constructed from (9) by holding real output and demand in each sector at their common base year levels and by dividing by  $y_1^s$ . Thus

$$P_{1t}^* = \sum_{i=1}^{n+m-1} w_i p_{it}, \quad (10)$$

where  $\sum_i w_i = 1$  holds through rearrangement of equation (1) and where we explicitly introduce the observation subscript  $t$  with  $i = n + 1, n + 2, \dots, n + m - 1$  weights,  $w_i$  being negative.  $P_{1t}^*$  can be estimated as a function of  $k = 1, 2, \dots, l$  regressors  $z_{kt}$  contained in  $Z_t$

$$\begin{aligned} P_{1t}^* &= P_1^*(Z_t) \\ &= a + \sum_{k=1}^l b_k z_{kt} + e_t, \end{aligned} \quad (11)$$

where  $e_t$  is a random disturbance term.

We wish to estimate expenditure price equations that are homogeneous of degree one in all production prices and explicitly recognize the individual expenditure sector market relationships which determine  $P_{1t}^*$ . Therefore we write

$$p_{it}^d = p_i(p_{2t}^s, p_{3t}^s, \dots, p_{mt}^s, P_1^*(Z_t)). \quad (12)$$

Homogeneity is maintained by constructing a fixed weight deflator gross of all production prices:

$$P_{2t}^* \equiv \sum_{i=1}^n v_i p_i^d \quad (13)$$

where the two fixed weight deflators are linked through the identity

$$P_{2t}^* = P_{1t}^* + \sum_{i=1}^n (v_i - w_i)P_i - \sum_{i=n+1}^{n+m-1} w_i P_i. \quad (14)$$

Each final demand expenditure price is then estimated as a weighted share of  $P_{2t}^*$  in the form

$$\begin{aligned} \frac{v_i p_{it}^d}{P_{2t}^*} &= \alpha_{0i} + \alpha_{1i} \frac{p^s_{2t}}{P_{2t}^*} + \alpha_{2i} \frac{p^s_{3t}}{P_{2t}^*} + \cdots + \alpha_{m-1,i} \frac{p^s_{mt}}{P_{2t}^*} \\ &+ \sum \beta_{ki} z_{kt} + \varepsilon_{it}, \end{aligned} \quad (15)$$

which over all  $i$  form a singular equation system with the properties

- (i)  $\sum_i \alpha_{0i} = 1$ ,
- (ii)  $\sum_i \alpha_{ji} = 0$  for any  $j$ th relative output price,
- (iii)  $\sum_i \beta_{ki} = 0$  for any  $z_k$ ,
- (iv)  $\sum_i \varepsilon_{it} = 0$  at any time  $t$ .

The singular equation system summing constraints guarantee that the effects of changes in expenditure prices are distributed totally across all  $n$  final demand sectors (property 16i) by sector weight without leakage at any point in time  $t$  (property 16iv). Final demand sector prices are affected by a production sector price, or the cost function determinants thereof, to a degree different from their weighted relative proportion of the total, which suggest nonzero  $\beta_{ki}$  or  $\alpha_{ji}$ s. Properties 16ii and 16iii maintain the consistency of the total of weighted average final demand prices while allowing for some deviation.

For any nonzero set of  $\beta_{ki}$ s it can be shown that the weighted share prices respond more or less than their share weight within the system to the estimated impacts of some  $z_k$ , on the total of the system. Homogeneity guarantees

$$\frac{\partial P_{2t}^*}{\partial z_k} = \sum_{i=1}^n v_i \frac{\partial p_{it}^d}{\partial z_k} = \sum_{i=1}^n v_i z_k \quad (17)$$

and similarly for the impacts of the other value added price effects estimated within the system

$$\frac{\partial P_{2t}^*}{\partial p_j^s} = \sum_{i=1}^n v_i \frac{\partial p_{it}^d}{\partial p_j^s} = \sum_{i=1}^n v_i \eta_{ji}, \quad (18)$$

The relationship between homogeneity guaranteed by (17) and (18) and properties (16iii) and (16ii), respectively, can be shown to be

$$\eta_{ki} = \frac{\beta_{ki} + v_i \hat{b}_k}{v_i} \quad (19)$$

and

$$\eta_{ji} = \frac{\sum_i x_i^d - y_j^s}{y_j^s} \hat{\alpha}_{ji} + v_i, \quad (20)$$

where  $\hat{b}_k = \partial P^*/\partial z_k$  estimated from (11). It is easily verified that the partial elasticities in either (19) or (20) totally exhaust the effects stemming from the sources of value added,  $\sum_i \eta_{ki} = \hat{b}_k$  and  $\sum_i \eta_{ji} = \sum_i v_i$ ,

$$\sum_i \beta_{ki} = \sum_i v_i (\eta_{ki} - b_k) = 0, \quad (21)$$

and

$$\sum_i \hat{\alpha}_{ji} = \frac{y_j^s}{\sum_i x_i^d - y_j^s} \sum_i (\eta_{ji} - v_i) = 0. \quad (22)$$

The condition which satisfies (22) is  $\eta_{ji} = d_{ji}$ , where the  $d_{ji}$  sum to unity across  $i$  by construction in (6).

Before exploring estimable forms of (11) and the system (15), it is useful to evaluate the necessary summing constraint properties in the context of the techniques we use for estimation. Specifically, it is necessary to estimate the system (15) under conditions that (a) impose stochastic restrictions on the  $\alpha_{ji}$  due to collinearity of the  $p_{ji}^s$  with elements of  $Z$ , (b) impose a distributed lag on the  $\beta_{ki}$  in order to capture meaningfully the lagged adjustment processes, and (c) estimate an iterative autoregressive process for the error disturbances across the system.

Conditions (a) and (b) can be specified using the mixed estimator of Theil (1963). Prior knowledge is available from the input-output derived  $d_{ji}$  coefficients which can be used in place of the  $\eta_{ji}$  in (20). These and the smoothness priors (Shiller 1973) on the lagged adjustment process form the set of stochastic restrictions. Condition (c) may be imposed by specifying a first- (or higher-) order autoregressive process so long as the process is universal across the system (Berndt and Savin 1975). Let  $y^i$  be the  $T \times 1$  vector of observations on  $v_i p_{it}^d / P_{2t}^*$  and  $X$  be the  $T \times q$  matrix of observations on the regressors on the right-hand side of (15). These would include the unit vector  $u$ , the  $m - 1$  relative value added prices, and the  $l$  contemporaneous and lagged values of  $z_k$ , each with lag length  $\lambda_k$ . Hence

$q = 1 + (m - 1) + \Sigma \lambda_k$ . The basic relationship to be estimated for the  $i$ th relative price equation is thus

$$Y^i = XB^i + \varepsilon^i, \quad (23)$$

which may be augmented by the relationship

$$r^i = RB^i + v^i, \quad (24)$$

where  $R$  is the matrix containing the restrictions on the  $\alpha_{ji}$ . The restrictions take the form  $\Sigma_i \alpha_{ji} = \Sigma_i r_j^i = 0$  and  $\Sigma_i r_k^i = 0$ , the latter being the non-Bayesian operational form of the Shiller lag. Errors associated with the stochastic restrictions are contained in the vector  $v^i$ . A first-order autoregressive process on  $Y_t^i$  and  $X_t^i$  forms the transformation

$$\begin{aligned} Y_t^{*i} &= Y_t^i - \rho Y_{t-1}^i, \\ X_t^{*i} &= X_t^i - \rho X_{t-1}^i. \end{aligned} \quad (25)$$

and the estimable form for (16) is

$$\begin{bmatrix} Y^{*i} \\ kr^i \end{bmatrix} = \begin{bmatrix} X^* \\ kR \end{bmatrix} B^i + \begin{bmatrix} \varepsilon^{*i} \\ kv \end{bmatrix} \quad (26)$$

Thus  $B^i$  may be estimated as

$$\hat{B}^i = (X^{*'} X^* + k^2 R' R)^{-1} X^{*'} Y^{*i}, \quad (27)$$

which will satisfy the properties (16) when

$$\sum_i \hat{B}^i = \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \quad \sum_i \varepsilon^{*i} = 0. \quad (28)$$

Let  $X^*$  and  $R$  be partitioned into

$$\begin{aligned} X^* &= \begin{bmatrix} u^* & \vdots & \tilde{X} \end{bmatrix} \\ R &= \begin{bmatrix} \theta & \vdots & \tilde{R} \end{bmatrix}, \end{aligned}$$

with  $u^*$  being a vector with elements  $(1-\rho)$  and  $\theta$  the null matrix conformable to row dimensions of  $R$  and column dimensions of the nonrestricted parameters in  $B^i$ ,  $\tilde{X}^*$  appropriately arranged. Equation (27) can then be written as

$$\hat{B}^i = \begin{bmatrix} T & u^{*'} \tilde{X}^* \\ \tilde{X}^{*'} u^* & \tilde{X}^{*'} \tilde{X}^* + k^2 \tilde{R}' \tilde{R} \end{bmatrix}^{-1} \begin{bmatrix} u^* \\ \tilde{X}^* \\ kR \end{bmatrix} \begin{bmatrix} Y^{*i} \\ kr^i \end{bmatrix}, \quad (30)$$



where  $T = u^{*'}u^*$ . Summing across  $n$  equations in the system where  $\Sigma_i Y^{*i} = 1$  by construction and  $\Sigma_i r^i = 0$  by design,

$$\Sigma \hat{B}^i = \begin{bmatrix} T & u^{*'}\tilde{X}^* \\ \tilde{X}^{*'}u^* & \tilde{X}^{*'}\tilde{X} + k^2\tilde{R}'\tilde{R} \end{bmatrix}^{-1} \begin{bmatrix} T \\ \tilde{X}^{*'}u^* \end{bmatrix}. \quad (31)$$

Using the rule for a partitioned inverse (Theil 1977, p. 18),

$$\begin{aligned} \Sigma \hat{B}^i &= \begin{bmatrix} \frac{1}{T} + \frac{1u^{*'}}{T} & \tilde{X}^*W\tilde{X}u^* & \frac{1}{T} - \frac{1}{T}u^{*'}\tilde{X}^*W \\ -W\tilde{X}^{*'}u^* & \frac{1}{T} & W \end{bmatrix} \begin{bmatrix} T \\ \tilde{X}^{*'}u^* \end{bmatrix} \\ &= \begin{bmatrix} 1 + \frac{1}{T}u^{*'}\tilde{X}^*W\tilde{X}^{*'}u^* - \frac{1}{T}u^{*'}\tilde{X}^*W\tilde{X}^{*'}u^* \\ -W\tilde{X}^{*'}u^* + W\tilde{X}^{*'}u^* \end{bmatrix} \\ &= \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \end{aligned} \quad (32)$$

where  $W = \tilde{X}^{*'}\tilde{X}^* + k^2\tilde{R}'\tilde{R} - \tilde{X}^{*'}u^*u^{*'}\tilde{X}^*/T$ .

Now for any  $i$ th relative price equation at time  $t$

$$\begin{aligned} \hat{\varepsilon}_t^i &= Y_t^i - \hat{Y}_t^i \\ &= (Y_t^i - \rho Y_{t-1}^i) - \left[ (1 - \rho)\hat{\beta}_{0i} - \sum_{k=1}^{q-1} \hat{\beta}_{ki}(X_{kt} - \rho X_{k,t-1}) \right]. \end{aligned} \quad (33)$$

Summing across all  $i$  equations at time  $t$

$$\begin{aligned} \sum_{i=1}^n \hat{\varepsilon}_t^i &= \sum_{i=1}^n (Y_t^i - \rho Y_{t-1}^i) - \sum_{i=1}^n (1 - \rho)\hat{\beta}_{0i} - \sum_{i=1}^n \sum_{k=1}^{q-1} \hat{\beta}_{ki}(X_{kt} - \rho X_{k,t-1}) \\ &= (1 - \rho) - (1 - \rho) - 0 \\ &= 0, \end{aligned} \quad (34)$$

where  $\Sigma Y^i = 1$  by construction;  $\Sigma_i \hat{\beta}_{0i} = 1$ ,  $\Sigma_i \Sigma_k \hat{\beta}_{ki} = 0$  from above.

In general these summing constraints will hold for any estimation technique of data transformation iff  $\Sigma Y^i = 1$  and  $X$  is universal throughout the system (Denton 1978).

### 3. ESTIMATION RESULTS

A singular equation system of relative prices applied to the MPS model includes four value-added production output deflators and ten final demand expenditure price deflators. In order to display the estimable form of this relative price system, we begin with a restatement of the identity in (2) expressed in terms of the disaggregated economic accounts of the model

$$\begin{aligned} Y_{nf}P_{nf} + Y_fP_f + Y_hP_h + Y_mP_m &= X_{cnd}P_{cnd} + X_{cd}P_{cd} + X_{rs}P_{rs} \\ &+ X_{pd}P_{pd} + X_{ps}P_{ps} \\ &+ X_{sc}P_{sc} + X_{so}P_{so} + X_{fc}P_{fc} + X_{fo}P_{fo} + X_{ex}P_{ex}. \end{aligned} \quad (35)$$

Value-added quantity and price is disaggregated into nonfarm (nf), farm (f), household and institutional (h), and imported (m) production sectors. Final demand quantity and price are disaggregated into ten sectors. These are consumer nondurables (cnd), consumer durables (cd), residential structures (rs), in producer durables (pd), producer structures (ps), state and local government expenditures on construction (sc) and other goods (so), federal government expenditures on construction (fc) and other goods (fo), and exports (ex).

The nonfarm business sector output price is modeled differently than the farm, household and institutional, and import output prices, as indicated in (9) above. It is given by

$$\begin{aligned} P_{nf} &= \frac{X_{cnd}}{Y_{nf}} P_{cnd} + \frac{X_{cd}}{Y_{nf}} P_{cd} + \frac{X_{rs}}{Y_{nf}} P_{rs} + \frac{X_{pd}}{Y_{nf}} P_{pd} \\ &+ \frac{X_{ps}}{Y_{nf}} P_{ps} + \frac{X_{sc}}{Y_{nf}} P_{sc} + \frac{X_{so}}{Y_{nf}} P_{so} \\ &+ \frac{X_{fc}}{Y_{nf}} P_{fc} + \frac{X_{fo}}{Y_{nf}} P_{fo} + \frac{X_{ex}}{Y_{nf}} P_{ex} \\ &- \frac{Y_f}{Y_{nf}} P_f - \frac{Y_h}{Y_{nf}} P_h - \frac{Y_m}{Y_{nf}} P_m. \end{aligned} \quad (35)$$

From (35) we can construct a nonfarm business fixed weight deflator,  $P_{nf}^*$ , using constant base year (1972) values for the real magnitudes of the expenditure and production sectors,

$$\begin{aligned} P_{nf}^* &\equiv w_{cnd}P_{cndt} + w_{cd}P_{cdt} + w_{rs}P_{rst} + w_{pd}P_{pdt} + w_{ps}P_{pst} \\ &+ w_{sc}P_{sct} + w_{so}P_{sot} + w_{fc}P_{fct} + w_{fo}P_{fot} \\ &+ w_{ex}P_{ext} + w_fP_{ft} + w_hP_{ht} + w_mP_{mt} \end{aligned}$$

$$\equiv \sum_{i=1}^n w_i P_{it}; \quad \sum_{i=1}^n w_i = 1. \quad (36)$$

The specification used to estimate (36) follows a markup over long-run minimized average cost in a less than perfectly competitive market. Theoretical derivation for this specification is found in Ando (1972), DeMenil and Enzier (1970), and later in Howe (1976) and Thurman (1979). The estimated form of the equation for  $P_t^*$  is

$$\begin{aligned} \ln P_{t,T}^* = & -0.0047 + 0.9810(L_6) \ln PL_t - 1.00(L_8) \ln OMH_t \\ & + 0.4248(L_6)UR_t + 0.0492(L_6) \ln P_t^* \\ & + 0.0284(L_4) \ln P_{et} - 0.1515(L_4) \Delta \ln P_{mt} - 0.0047D_{ct}, \end{aligned} \quad (37)$$

where  $L_{\lambda-1}$  is a distributed lag operator of  $\lambda$  periods (including the current period), and where long run minimized average cost is proxied by unit labor costs—wages (PL) divided by output per manhour (OMH)—with a steady-state coefficient of unity. Arguments in the markup function include the inverse of a geometric average of unemployed resources in the economy (UR), competing goods price effects from foreign exchange rate adjusted foreign prices ( $P_t^*$ ), and weighted average wholesale energy prices ( $P_e$ ), the latter two relative to moving average domestic consumption prices. Changes in raw material prices ( $P_{rm}$ ) are included to account for the temporary negative reaction of production prices to surges in input prices which were not included in the construction of value added. A dummy variable ( $D_c$ ) is specified for the price controls period 1972Q2–1975Q2. The sample period for estimation was 1956Q1–1978QIV. Results of estimating the value-added price equations are provided in Table 1. A first-order autoregressive process was assumed with  $\rho$  estimated iteratively by the Cochrane–Orcutt procedure.

For the relative price system of final demand prices a fixed weighted average price index gross of value-added input prices was constructed in the form of (13):

$$\begin{aligned} P_{t,T}^* \equiv & \frac{X_{cnd}}{T} P_{cndt} + \frac{X_{cd}}{T} P_{cdt} + \frac{X_{rs}}{T} P_{rst} + \frac{X_{pd}}{T} P_{pdt} \\ & + \frac{X_{ps}}{T} P_{pst} + \frac{X_{sc}}{T} P_{sct} + \frac{X_{so}}{T} P_{sot} \\ & + \frac{X_{fc}}{T} P_{fct} + \frac{X_{fo}}{T} P_{fot} + \frac{X_{exn}}{T} P_{ext} \\ \equiv & \sum_{i=1}^n v_i P_{it}; \quad \sum_{i=1}^n v_i = 1, \end{aligned} \quad (38)$$

Table 1: Estimated Coefficients for P\*1 Equation

Constant	-0.0046728 (1.17)	UR $t = 0$	0.15771 (6.36)	ln Pe $t = 0$	.015466 (2.35)
		$t = -1$	0.1183 (6.83)	$t = -1$	.010614 (2.92)
ln PL					
$t = 0$	0.31859 (15.18)	$t = -2$	0.080689 (5.70)	$t = -2$	.0046903 (1.11)
$t = -1$	0.25656 (27.42)	$t = -3$	0.048045 (3.58)	$t = -3$	-.0023214 (0.69)
$t = -2$	0.19505 (24.54)	$t = -4$	0.022649 (1.85)	Sum	.028449 (4.26)
$t = -3$	0.13389 (12.37)	$t = -5$	0.0048991 (0.51)		
$t = -4$	0.07103 (6.49)	$t = -6$	-0.0075044 (-0.96)		
$t = -5$	0.0068213 (0.61)	Sum	0.42478 (6.10)	$\Delta$ ln Prm $t = 0$	-.077904 (-4.50)
Sum	0.98194 (154.47)	$t = 0$	.0033994 (0.36)	$t = -1$	-.047495 (-5.51)
		$t = -1$	.0083896 (2.00)	$t = -2$	-.023724 (-2.40)
		$t = -2$	.012385 (2.87)	$t = -3$	-.0023955 (0.36)
ln OMH		$t = -3$	.01425 (2.57)	Sum	-.15152
$t = 0$	-0.23631 (-10.29)	$t = -4$	.01033 (2.05)	CONTROL	-.00467 (-1.17)
$t = -1$	-0.20029 (-16.03)	$t = -5$	.00044592 (0.59)		
$t = -2$	-0.16835 (-18.91)	Sum	.049203 (19.19)		
$t = -3$	-0.13972 (-13.86)	CONTROL= 0.0	1956QI-1971Q4		
		= 0.42	1972 Q1	= 0.94	1974 Q1
$t = -4$	-.11269 (-9.77)	0.63	Q2	0.64	Q2
		0.81	Q3	0.44	Q3
		0.98	Q4	0.30	Q4
$t = -5$	-.083118 (-7.09)	1.11	1973 Q1	0.21	1975 Q1
		1.17	Q2	0.14	Q2
		1.11	Q3	0.00	1975 Q3
$t = -6$	-0.048874 (-4.77)	1.00	Q4		
$t = -7$	-0.010705 (-1.04)	$\bar{R}^2 = 0.99989$	Se = 0.0025834	DW = 1.95	

Table 1: *Continued*

Sample Period: 1956QI-1978QIV	
Sum	-1.00 (-125.27)
Shiller Lag Constraints	
$a_i$ )	first degree Shiller smoothness priors, $k = 0.1$ , endpoint restriction
$b_i$ )	first degree Shiller smoothness priors, $k = 0.05$ , endpoint restriction, lag sum constrained to 1.0 with $k = 0.05$
$c_i$	first degree Shiller smoothness priors, $k = 0.05$ , endpoint restriction, lag sum constrained to -1.0 with $k = 0.1$
$d_i$ )	first degree Shiller smoothness priors, $k = 0.1$ , endpoint restriction
$f_i$ )	first degree Shiller smoothness priors, $k = 0.1$ , endpoint restriction, lag sum constrained to 0.05 with $k = 1$
$g_i$ )	first degree Shiller smoothness priors, $k = 0.1$ , endpoint restriction, lag sum constrained to 0.03 with $k = 0.1$

where  $T$  is the sum of real 1972 based final demand expenditures.  $P_2^*$  may be defined as the gross domestic sales fixed weight deflator and can be shown to have the following relationship to its nonfarm value added counterpart  $P_1^*$ :

$$P_2^* = P_1^* + \sum_{i=1}^q (v_i - w_i)P_{it} + v_{10}P_{ext} - w_{10}P_{ext} - \sum_{i=1}^{13} w_i P_{it}. \quad (39)$$

Equation (39) differs from the form of (13) since the agricultural ( $P_{exa}$ ) and service ( $P_{exs}$ ) components of the total export expenditure price deflator ( $P_{ex}$ ) are not determined by the markup over cost specification. Consistency in (39) is maintained through the approximation for  $P_{ex}$  in

$$P_{ext} = \omega_1 P_{ext} + \omega_2 P_{exat} + \omega_3 P_{exst} \quad (40)$$

which includes the nonagricultural export expenditure deflator ( $P_{exn}$ ) contained in (38) and where  $\sum_{i=1}^3 \omega_i = 1$ . The weights for (39) are given in Table 2.

The system of relative final demand prices (15) is given by

**Table 2: Fixed Weight Values for  $P^*2$  and  $P^*1$  By Sector**

$i^a$	$P_i$	$V_i$	$W_i$
1	$P_{cnd}^*Ta^b$	0.58250	0.65107
2	$P_{cd}^*Ta^b$	0.10422	0.11659
3	$P_{rs}$	0.05806	0.00490
4	$P_{pd}$	0.06958	0.07778
5	$P_{ps}$	0.03984	0.04453
6	$P_{sc}$	0.02471	0.02762
7	$P_{so}$	0.03499	0.03906
8	$P_{fc}$	0.00415	0.00464
9	$P_{fo}$	0.04464	0.04990
10	$P_{exn} \cdot P_{ex}$	0.03735	0.07610
11	$P_m$		-0.03356
12	$P_f$		-0.07943
13	$P_h$		-0.03898

<sup>a</sup>For explication of variables, see text following equation (35).

<sup>b</sup>Ta = ( $X_{cnd}$  \$ +  $X_{cd}$  \$) - Tx/( $X_{cnd}$  \$ +  $X_{cd}$  \$), where Tx are nominal federal indirect business taxes, purged from Pcnd and Pcd to reflect the fact that excise taxes add nothing to value added although they are directly passed on to prices purchasers pay for final goods. In earlier versions of estimating the model's price system (see Thurman (1977)) indirect business taxes were tried as regressors with perverse results. The estimates retained most of the burden of excise taxes on the producers without any passthrough to ultimate consumers. Since this is a concept difficult to accept, the two consumption deflators were adjusted before estimation for federal indirect business taxes.

$$\begin{aligned}
 \frac{v_i P_{it}}{P_{2t}^*} = & a_{0i} + a_{1i} \frac{P_{mt}}{P_{2t}^*} + a_{2i} \frac{P_{ft}}{P_{2t}^*} + a_{3i} \frac{P_{ht}}{P_{2t}^*} \\
 & + b_i(L_2)UOS_t + c_i(L_5)CU_t + d_i(L_5) \ln OMH_t \\
 & + e_i(L_8) \ln PL_t + f_i \frac{1}{TT} + g_i(L_6)P_{ft}^* \\
 & + h_i D_{ct} + \varepsilon_{it}
 \end{aligned} \tag{41}$$

Coefficients for the  $C$  matrix, obtained from the Wharton Annual and Industry Forecasting Model (Preston 1972), were used to calculate the  $\eta_{ji}$  elasticities in (20) for the import, farm, household, and institutional output sectors. These approximate the proportional output of each production sector consumed within each  $i$ th final demand sector and are

$$\begin{aligned}
\eta_{1i} &= (0.336, 0.168, 0.1029, 0.176, 0.034, -0.042, \\
&\quad 0.0059, 0.0176, 0.1885, 0.0401) \\
\eta_{2i} &= (0.0828, 0.014, 0.026, 0.034, 0.017, 0.011, \\
&\quad 0.023, 0.001, 0.02, 0.026), \\
\eta_{3i} &= (1, 0, 0, 0, 0, 0, 0, 0, 0, 0). \tag{42}
\end{aligned}$$

Using the  $\eta_{ji}$  coefficients, the expected price sector coefficients relative to each value-added price deflator from (22) are

$$\begin{aligned}
a_{1i} &= (-0.0175, 0.0049, -0.0021, 0.0081, \\
&\quad -0.00037, -0.0015, -0.0021, 0.001, \\
&\quad 0.0108, -0.0013), \\
a_{2i} &= (0.0078, -0.0026, -0.00093, -0.0010, \\
&\quad -0.00067, -0.0004, -0.00034, -0.00009, \\
&\quad -0.00072), \\
a_{3i} &= (0.0152, 0.0036, -0.002, -0.0024, -0.0014, \\
&\quad -0.00085, -0.0012, -0.00014, -0.0015).
\end{aligned}$$

It was necessary to impose these values on the system due to the substantial collinearity between the value-added price regressors and the regressors in (37), which were included in the system estimation (41). The system summing constraints require that the vectors  $a_{ji}$  sum to zero.

The functional form of (41) differs in several minor respects from the markup over cost specification used for  $p_1^*$ . Standard unit labor costs—wages (PL) adjusted by output per manhour (OMH)—form the same approximation to the long-run cost function. In the markup function the relative price equations were found to be more highly sensitive to demand pressure variables in the form of the ratio of unfilled orders to shipments of producers durable equipment (UOS) and capacity utilization (CU) than the term used for unemployed resources in the equation for  $P^*$ . Competing goods price effects in the relative price system were estimated for foreign consumption price ( $P^*$ ), which was adjusted by the exchange rate.  $D_c$  represents a binary dummy for the price control period 1972Q2–1975Q2. The inverse of the time trend (TT) accounts for any trend pattern in the growth of relative sector prices.

The sample period for estimating the relative price system (41) was 1960Q2–1978Q2. A first-order autoregressive process was assumed for the system and again mixed estimation was used. A summary of the estimates for the relative price system is displayed in Table 3.

Of fundamental importance to the analysis is how the relative price system coefficients and the final elasticities of the expenditure prices are related. While equations (19) and (20) provide guidance for these

Table 3: Summary of Results of Relative Price System<sup>a</sup>

	PCND	PCD	PRS	PPD	PPS	PSC	PSO	PFC	PFO	P <sub>exn</sub>
Constant	0.631*	0.327*	-0.203	0.159	-0.002	0.079*	-0.039	0.021*	0.102	-0.078
a <sub>1</sub>	-0.014*	-0.006	-0.002	0.002	0.004*	0.001	-0.001*	0.001*	0.009*	0.002
a <sub>2</sub>	0.006*	-0.004*	0.001*	-0.003*	0.001*	0.0004	-0.0004	0.0003*	-0.001	-0.0009
a <sub>3</sub>	0.015*	-0.004*	-0.002*	-0.003*	0.001*	-0.0008*	-0.001*	-0.0001	-0.002*	-0.001*
b <sub>1</sub>	0.013	-0.015*	0.004	-0.016*	0.008*	0.004	-0.003	-0.004	0.006	-0.001
b <sub>2</sub>	-0.013	-0.0008	-0.004	0.005	0.002	-0.003	0.003*	-0.0007	-0.010	0.021*
Σx <sub>i</sub>	0.009	-0.080*	0.036*	-0.035	0.015	0.0003	0.010	-0.002	0.021	0.026*
Σd <sub>i</sub>	-0.019	-0.66*	0.034*	-0.028*	0.029*	0.006	0.012*	-0.001	0.002	0.032*
Σe <sub>i</sub>	0.027	0.128*	0.0001	0.059	-0.072*	-0.040*	-0.006	-0.003	-0.032	-0.062*
f	0.0002	-0.002*	0.001*	-0.001	0.002*	0.001*	-0.0002	0.0001*	0.0000	-0.001*
Σg <sub>i</sub>	0.015	0.004	-0.009*	0.001	-0.008*	0.003*	0.001*	-0.001*	-0.012*	0.006*
h	-2.044	0.524	3.036*	0.306	-0.553	-1.386*	0.818*	-0.283*	-1.168	0.756
Sep <sup>b</sup>	0.0012	0.0008	0.0004	0.0006	0.0004	0.0003	0.0002	0.00004	0.0006	0.0005
Sep <sup>c</sup>	0.215	0.723	0.670	0.930	0.984	1.047	0.443	1.022	1.185	1.258
R <sup>2</sup>	0.915	0.993	0.962	0.969	0.989	0.984	0.946	0.980	0.385	0.971
DW	1.59	1.09	1.96	0.935	1.141	1.220	2.049	2.56	2.709	1.67
ρ	0.3271	0.3271	0.3271	0.3271	0.3271	0.3271	0.3271	0.3271	0.3271	0.3271

<sup>a</sup>(\*) indicates that the coefficient or lag distribution was significant at the 5% level. A more detailed set of results is available upon request.

<sup>b</sup>Standard error of the regression.

<sup>c</sup>Standard error of the regression normalized with respect to the price level.



elasticities, the translog form used in the relative price system makes calculating them difficult. Consequently, we have derived the elasticities by using multiplier results from the price system coded within the MPS model. The multipliers are calculated from a static single equation simulation and are expressed as ten quarter percentage differences from historical values. With a maximum lag length in the system of nine quarters and including the autoregressive correction process, all lagged effects are captured in the multipliers by the end of the simulation. These results are tabulated in Table 4.

In general we expect the direction of the simulated elasticities for the final demand expenditure prices in Table 4 to follow those for  $P_1^*$  and  $P_2^*$ , only with differing magnitudes for those relative price equations that have significant nonzero coefficients. These expectations are realized throughout most of the relative price system with several notable exceptions. The response of normalized durable goods price deflators for both consumers ( $P_{cd}$ ) and producers ( $P_{pd}$ ) exhibit significantly less than unitary response to normalized unit labor costs (the sum of simulated effects for PL and OMH). These same two deflators also respond perversely to simulated shocks from farm price ( $P_f$ ) and the two demand pressure variables (CU and UOS). It should be noted that the remainder of the price deflator responses to these impacts compensates for these errors due to the nature of the "adding-up" constraints of the full system. Durable goods prices therefore need to be carefully monitored in multiplier analysis, which uses these shocks.

#### 4. DYNAMIC SIMULATION AND MULTIPLIER ANALYSIS

We now turn our attention to the stability properties of the estimated relative price system. We will discuss these properties from the perspective of the simulation tracking behavior of the system. The approach that we take in our dynamic simulations are by necessity somewhat asymmetrical. The estimated equations presented in section 3 represent only a portion of the much larger system. The distinction between simulated partial equilibrium analysis and general equilibrium analysis is paramount here. The full MPS model structure is capable of simulating all of the typical domestic macroeconomic responses to price shocks. In the simulation analysis of the price sector alone—the results of which are discussed in section 3—we examine the price sector's partial equilibrium responses to other model variables which are coded directly within the price system structure. Although these results are useful in allowing us to evaluate the "correctness" of the sector's responses to key determining variables, the key feedbacks and full model simultaneous interactions are

Table 4: Simulated Price System Elasticities [%  $\Delta$  PP/%  $\Delta$  Z]

	$P_f$	$P_m$	$P_b$	P	OMH	CU	UOS	$P_f^*$	w
$P^*1$	-0.0	-0.0	-0.0	0.92	-1.07	0.10	0.00	0.04	1.0
$P^*2$	0.02	0.10	0.03	0.89	-1.02	0.10	0.01	0.08	1.0
PCND	0.03	0.06	0.06	0.84	-0.91	0.10	0.01	0.09	0.583
PCD	-0.02	0.10	-0.0	0.11	0.34	-0.58	-0.11	0.12	0.104
PRS	0.05	0.07	0.0	1.65	-1.2	0.57	0.01	-0.10	0.058
PPD	-0.2	0.13	-0.0	0.43	-0.18	-0.31	-0.10	0.12	0.07
PFS	0.05	0.23	0.0	1.72	-2.8	0.40	0.16	-0.07	0.039
PSC	0.03	0.09	-0.0	1.00	-2.4	0.10	0.03	0.21	0.025
PSO	0.01	0.05	-0.0	1.19	-1.2	0.31	0.02	0.10	0.035
PFC	0.07	0.44	-0.0	0.70	-1.86	-0.16	-0.14	0.03	0.44
PFO	-0.01	0.40	-0.0	1.06	-1.92	0.49	-0.02	-0.06	0.004
$P_{\text{exp}}$	-0.004	0.14	-0.0	1.43	-2.18	0.53	0.31	0.24	0.037

missing. What we may observe as correct and stable static partial equilibrium price sector behavior may potentially introduce implausible or explosive dynamic properties to the model as a whole. Hence, in the presentation of dynamic simulation properties and multiplier results, the limits imposed by these omissions must be considered. By selecting different levels of endogenous variable determination within the MPS model simulation algorithm, these limits can be observed and the interpretations of simulation results can be qualified accordingly.

Before we evaluate the dynamic stability properties of the model used in this study, we note that the structure and size of the MPS model precludes the evaluation of the statistical significance of the model's dynamic properties. These could in principle be derived from the model's estimated structural coefficients and their covariances. However, procedures that evaluate the asymptotic distributions of impact and dynamic multipliers and the forecast accuracy of large econometric models (Bianchi et al 1981; Brissimis and Gill 1978; Dhrymes 1973; and Schmidt 1973) are computationally expensive and are difficult if not impossible to apply to models the size and structure of the MPS model. Most of these procedures assume a simultaneous method of estimating the structural model system and the absence of nonlinearities that restrict the necessary computations. Where numerical examples are provided in these studies, they are based on Klein's Model I of the U.S. economy, which is a six-equation model containing three behavioral equations, three identities, and a maximum variable lag length of two in the endogenous variable matrix. Computational requirements for even such a compact model structure are substantial.

The MPS model, like most econometric models used for policy analysis, is of considerably larger size and more complicated structure than Klein's Model I. These larger models are inherently nonlinear and are primarily estimated by single equation methods. Consequently the derivation of the system's reduced form and its covariance matrix is quite problematic.

The limits imposed by the size and structure of large econometric models do not necessarily preclude the analysis of the models stability properties however. Schmidt (1973) has examined small sample evidence of the dynamic simulation properties of such models using stochastic simulation techniques and McCarthy (1972) and Fair (1978) have demonstrated the usefulness of such techniques to evaluate the predictive accuracy of the Wharton and Fair econometric models, respectively.

Four summary statistics are used to evaluate the dynamic simulation performance of the estimated price equations for the MPS model. These

summary statistics are expressed in terms of percentage root mean squared error (%RMSE) over the simulation period.

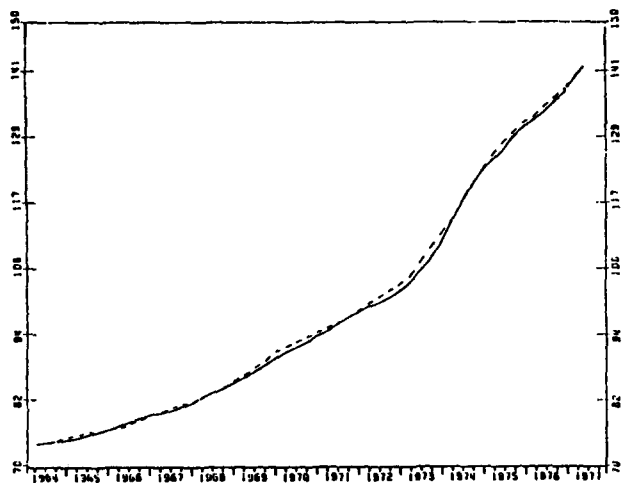
The summary statistics calculated are derived from four different types of dynamic simulations and are listed in Table 5. Single-equation dynamic percentage root mean square errors (%RMSES) represent a measure of the tracking ability of a coded equation where simulated values of the own lagged dependent variables are used in the calculation of the simulated equation with an autoregressive scheme. Single-sector dynamic percentage root mean square errors (%RMSEP) are calculated using simulated contemporaneous and lagged values of endogenous variables for the simultaneous price sector of the model.

The full-model dynamic percentage root mean square error ( $RMSEM_1$ ) is calculated from simulations of the full-model structure, as is the stochastic simulation error statistic ( $RMSES_2$ ). Both of these error calculations demonstrate the performance of the simulated price equation in conjunction with the dynamic tracking of the equations for the domestic model. In addition to the summary statistics we present plots of actual versus simulated prices during the estimating period. These are given in Figure 1 and 2. Other plots are available upon request.

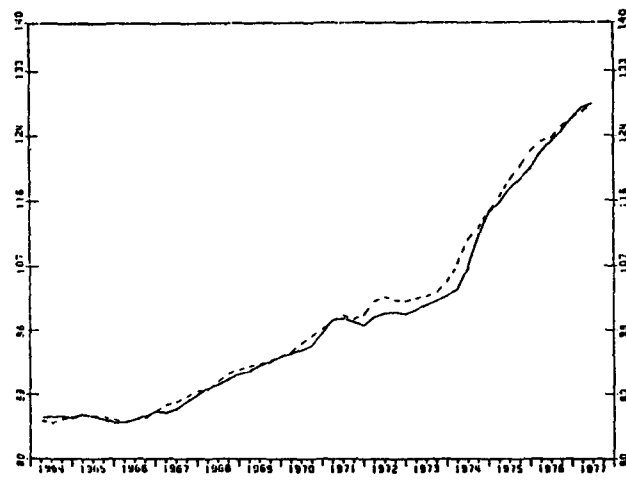
The domestic relative price system exhibits two troublesome periods in dynamic simulation that were not evidenced in the evaluation of the estimation results. These periods are approximately 1972Q2–1974Q2 and 1975Q2–1976Q3. In both periods the system oversimulates the actual values. The most likely source of those trouble periods is the simulated behavior of farm output prices ( $P_f$ ), which exhibited volatile behavior in these periods. Value-added price behavior in  $P^*_1$  underadjusts

**Table 5:** Dynamic Simulation Summary Statistics of Estimated Price Equations

		%RMSES	%RMSEP	% $RMSEM_1$	% $RMSEM_2$
1.	$P_{cnd}$	0.2	0.8	1.5	0.2
2.	$P_{cd}$	0.7	1.2	3.6	1.0
3.	$P_{RS}$	0.7	1.4	3.1	1.6
4.	$P_{pd}$	0.9	1.4	4.0	0.9
5.	$P_{ps}$	1.0	1.4	5.6	2.1
6.	$P_{so}$	0.4	1.0	3.8	0.8
7.	$P_{sc}$	1.1	2.3	1.8	0.8
8.	$P_{fo}$	1.2	1.4	3.7	2.3
9.	$P_{fc}$	1.1	1.2	2.3	2.3
10.	$P_{exn}$	1.2	2.7	5.5	2.9
11.	$P_f^*$	0.5	0.7	1.6	0.3
12.	$P_f^*$	0.3	0.5	1.4	0.4



**Figure 1.** Price Deflator for Consumer Nondurables, 1972 = 100. Solid line is actual; dashed line is simulated.



**Figure 2.** Price deflator for consumer durables (1972 = 100). Solid line is actual; dashed line is simulated.

for farm material prices, which stimulate well below the increase in farm prices so that  $P^*_1$  oversimulates. Gross sales prices in the relative price system must therefore distribute both the oversimulated  $P^*_1$  and under-simulated  $P_f$ . This inspection of the domestic relative price system tracking behavior indicates that the coded historical ratio treatment of agricultural price behavior needs to be replaced by a method of determination that includes the detailed market force determinants of this price.

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