Returns to scale and curvature in the presence of spillovers: Evidence from European countries

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January 2014

Abstract

Drawing on a recent development on the interpretation of spatial econometric models we extend two classic characteristics of production (returns to scale and diminishing marginal productivity of factor inputs) to the spatial case. In the context of a spatial translog production function we define own (i.e. direct), spillover (i.e. indirect) and total (i.e. direct plus indirect) returns to scale. The spatial production function gives rise to direct, indirect and total productions functions so we set out empirical checks to establish if these functions are concave. The ideas of spatial returns to scale and spatial concavity/convexity can easily be applied to other technical relationships (e.g. cost and distance functions) and other functional forms (e.g. Cobb-Douglas). We apply these ideas to aggregate production of European countries and find that the 2004 EU enlargement led to a sharp fall in direct, indirect and total returns scale across the EU.

Keywords: Spatial Returns to Scale; Spatial Concavity; Spatial Durbin Model.
JEL Classification: C23; D24

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1 Introduction

In this paper we introduce the idea of spatial returns to scale by drawing on a recent development in applied spatial econometrics. LeSage and Pace (2009) demonstrate that the parameters from a model which contain the spatial autoregressive variable cannot be interpreted as marginal effects. To address this problem they propose a method to calculate direct, indirect and total marginal effects.[1] Using this method we propose direct, indirect and total returns to scale in the context of a production function. Direct, indirect and total returns to scale can easily be calculated for other technical relationships (cost, standard and alternative profit, revenue, and input and output distance functions) so there considerable scope for wider application of the spatial returns to scale which we propose here. Direct returns to scale has the same interpretation as own returns to scale from a non-spatial production function. Indirect returns refers to the rate of increase in a unit's output following an increase in the factor inputs of neighbouring units. Total returns to scale is the rate of increase in a unit's output following an increase in its own factor inputs and its neighbours' factor inputs. Having estimated spatial production functions for European countries over the period $1990-2011$ using various spatial weights matrices, we calculate direct, indirect and total returns to scale.

The closest relatives to this paper fall into two categories: (i) empirical applications of New Economic Geography (NEG) and (ii) empirical growth models estimated using spatial econometric techniques.[2] There are a wide range of empirical applications of NEG to, for example, countries (Redding and Venables, 2004), Brazilian states (Fally *et al.*, 2010), Chinese prefectures (Hering and Poncet, 2010), U.S. counties (Hanson, 2005) and NUTS-2 European regions (Head and Mayer, 2006; Baltagi *et al.*, 2013). For a sample of local areas in Great Britain, Fingleton (2006) combines spatial econometric methods and an artificial nesting model to test unnested NEG and urban economic theories against one another. The urban economic theory is based on the benefits from good connections with producers in the service sector, where these linkages are better in urban areas where employment density is higher. Interestingly, Fingleton finds that the data supports urban economic theory over NEG. In essence NEG is a set of price and wage equations, where the vast majority of applied NEG studies estimate the wage equation because of its empirical tractability. In the wage equation the effect of geography is in terms of the effect of a region's market potential. Regions have different levels of market potential because of differences in access costs to its own and other regional markets (i.e. differences in

[1] The direct elasticity is interpreted in the same way as an own elasticity from a non-spatial model although the direct elasticity takes into account feedback effects (i.e. effects which pass through first order and higher order neighbors via the spatial multiplier matrix and back to the unit which initiated the change). An indirect elasticity is the change in the dependent variable for one particular unit following a change in an explanatory variable in all the other units. The total elasticity is the sum of the direct and indirect elasticities.

[2] See Fujita *et al.* (1997) for a textbook treatment of the theory of NEG.

transport costs), differences in the size of markets (i.e. differences in incomes between regions) and differences in competition within a region resulting in price differences between regions. In this paper, however, the effect of geography is explicitly related to neighbours' factor inputs.

The growth models which have been estimated using spatial econometric techniques are the: (i) neoclassical growth model (Solow, 1956) and various extensions, and (ii) Verdoorn's model.[3] A small number of studies use spatial econometric techniques in conjunction with an artificial nesting model to test the non-nested standard neoclassical growth model and NEG against each other (Fingleton, 2008; Fingleton and Fischer, 2010). In both studies the neoclassical growth model is augmented with a spatial error term. On the other hand, other studies have developed the neoclassical growth model by introducing spatial knowledge spillovers to the Solow residual (López-Bazo *et al.*, 2004; Egger and Pfaffermayr, 2006; Ertur and Koch, 2007; Pfaffermayr, 2009). As a result, familiar spatial econometric models such as the spatial Durbin model, which has a spatial autoregressive variable and spatial lags of the independent variables, are linear approximations of the reduced form TFP and convergence equations.

McCombie and Fingleton (1998), Fingleton and López-Bazo (2006), and Pfaffermayr (2009) observe strong empirical support for Verdoorn's model using data for NUTS-2 European regions. Fingleton (2001) also notes that Verdoorn's model is better suited to explaining regional growth patterns than the neoclassical growth model because it is consistent with some endogenous growth models and with some theories based on agglomeration economies from the urban economics literature. However, as Pfaffermayr (2009) notes it is not possible to construct an artificial nesting model to test the non-nested Verdoorn and neoclassical growth models against one another.[4] Notwithstanding the important contributions of the above spatial empirical growth models, the authors do not calculate direct, indirect and total marginal effects. The direct, indirect and total marginal effects associated with the spatial error term are not particularly informative as they relate to the disturbance. Here, however, the spatial autoregressive variable forms part of the fitted models so we can clearly interpret the spillovers which are at work because the direct, indirect and total marginal effects relate to the independent variables.

The direct, indirect and total returns to scale which we propose here can be calculated using any functional form (Cobb-Douglas, Fourier flexible, translog, Generalized McFadden, Generalized Leontief, etc.). In an empirical analysis where the focus is on direct, indirect and total returns to scale it is more illuminating to calculate these returns from a function with a flexible form. Accordingly, in the application to aggregate production of European countries rather than estimate a spatial Cobb-Douglas production function

[3] Verdoorn's law postulates that output growth in a region will lead to an increase in the regions's labour productivity because of increasing returns to scale.

[4] The reason is because independent variables in the artificial nesting model would include log differences of income and population, which together would make up the dependent variable.

which underpins the above spatial empirical growth models and where returns to scale are the same at every point in the sample, we estimate a spatial translog production function which allows returns to scale to vary over the sample.

The direct, indirect and total marginal effects collectively constitute three translog production functions. An issue which is related to direct, indirect and total returns to scale over the sample is the curvature of the associated translog production functions. This is an important issue because concavity of a non-spatial or direct translog production function would satisfy the curvature property and would thereby indicate diminishing marginal productivity of a unit's own factor inputs. We conduct empirical checks of the curvature of the fitted direct translog functions by adapting the approach which is used to check the curvature of a non-spatial translog function. Indirect and total translog functions have no such curvature properties but we conduct an empirical check of the curvature of these functions in a similar way. The same approach can be used to conduct an empirical check of the curvature of the direct, indirect and total translog functions for other technical relationships (cost, standard and alternative profit, etc.).

In the application to aggregate production of European countries we estimate spatial autoregressive and spatial Durbin translog production functions using sixteen inverse distance spatial weights matrices. Notwithstanding that there is some merit in the spatial autoregressive models, we prefer the spatial Durbin models. Our reasons for this preference are explained at various junctures in the remainder of the paper. In the first spatial weights matrix there is spatial interaction between all the countries in the sample. The other fifteen spatial weights matrices represent different subsets of the spatial interaction in the first matrix, namely interaction with a certain number of: (i) near neighbours, (ii) big import partners and (iii) big export partners. We are therefore able to examine whether the spatial returns to scale and spatial curvature change when the spatial interaction is limited to different subsets of countries in the sample *vis-à-vis* when there is spatial interaction between all the countries in the sample. To give an insight into the key empirical findings, we find that the 2004 EU enlargement led to a marked fall in direct, indirect and total returns to scale across the EU. It is evident from post 2004 returns to scale for the EU that the marked fall in 2004 had a persistent effect.

The remainder of this paper is organised as follows. In section 2 we set out the two specifications of the spatial translog production function which we estimate and then we explain the approach to calculate direct, indirect and total returns to scale. Section 3 discusses how we conduct empirical checks of the monotonicity and curvature of the fitted direct, indirect and total translog production functions. Section 4 is dedicated to the application where, among other things, we test for constant direct, indirect and total returns to scale. Section 5 concludes.

2 Spatial Translog Production Functions and Spatial Returns to Scale

Our starting point is the spatial Durbin Cobb-Douglas production which Ertur and Koch (2007) estimate. Moreover, LeSage and Pace (2009) make a compelling econometric case for the spatial Durbin model.[5] There are two strands to the econometric case which LeSage and Pace make in support of the spatial Durbin model. The first is based on their belief that the principal focus of spatial modelling should be the analysis of substantive global spillovers which relate to the spatial autoregressive term and not the global spillover of shocks which emanate from the spatial error term. The second strand concerns the unbiased parameters which the spatial Durbin model yields if the true data generating process is, among others, the spatial error model or the spatial autoregressive model.

The spatial Durbin translog production function for panel data which we estimate is as follows:

$$\begin{aligned}\ln y_{it} &= \kappa + \psi_t + \alpha_i + \rho_1 t + \rho_2 t^2 + TL\left(x_{it}\right) + z_{it}\vartheta + \sum_{j=1}^{N} w_{ij} TL\left(x_{jt}\right) + \\ &\quad \sum_{j=1}^{N} w_{ij} z_{jt} \upsilon + \lambda \sum_{j=1}^{N} w_{ij} \ln y_{jt} + \varepsilon_{it}, \qquad (1)\end{aligned}$$

where N is a cross-section of units; y_{it} is the output of the ith unit at time t; ψ_t is a time period effect; α_i is a fixed effect; t is a time trend; x_{it} is a $(1 \times M)$ vector of inputs; $TL(x_{it})$ $= \gamma' \ln x_{it} + \frac{1}{2} \ln x'_{it} \Theta \ln x_{it}$ represents the technology as the translog approximation of the log of the production function, where γ' is a vector of parameters and Θ is a matrix of parameters; λ is the spatial autoregressive parameter; w_{ij} is a known non-negative element of the $(N \times N)$ spatial weights matrix, W; z is a vector of variables and ϑ and υ are vectors of parameters, where the z variables, the spatial lags of the z variables and the spatial autoregressive term shift the production technology; ε_{it} is an i.i.d. disturbance for i and t with zero mean and variance σ^2. W captures the spatial arrangement of the cross-sectional units and also the strength of the spatial interaction between the cross-sectional units. As is standard the diagonal elements of W are set to zero. W must be specified

[5] Their case for the spatial Durbin model is at odds with the view that spatial model selection is a matter for LM test procedures. LeSage and Pace (2009) make the case for the spatial Durbin model because of concerns about the robustness of these tests to misspecification of the spatial dependence. Early LM tests for spatial error autocorrelation (Burridge, 1980) and spatial autoregression (Anselin, 1988) ignore the possibility of spatial autoregression and spatial error autocorrelation, respectively. Anselin *et al.* (1996) develop cross-sectional LM tests for spatial error autocorrelation and spatial autoregression where both tests are robust to misspecification of local spatial dependence but not global spatial dependence. Furthermore, there is a debate about whether to adopt a specific-to-general testing procedure (Florax, *et al.*, 2003), a general-to-specific approach (Mur and Angulo, 2009) or a mix of the two (Elhorst, 2012).

prior to estimation and is usually specified according to some measure of geographical or economic proximity.

In Eq. 1 we omit t and t^2 from the translog function to avoid perfect collinearity with the corresponding spatial lags but include t and t^2 outside the translog function to take account of technological change. Eq. 1 is strictly therefore a partial spatial Durbin model and the technological change in Eq. 1 is Hicks-neutral because it does not affect the balance of factor inputs in the production function. Hicks-neutral technological change, however, restricts the factor substitution possibilities. Alternatively, we could relax the assumption of Hicks-neutral technological change in Eq. 1 by retaining $\sum_{j=1}^{N} w_{ij} TL\left(x_{jt}\right)$ and replacing $TL\left(x_{it}\right)$ with $TL(x_{it}, t) = \gamma^{'} \ln x_i + \frac{1}{2} \ln x_i^{'} \Theta \ln x_i + \rho_1 t_i + \rho_2 t_i^2 + \phi^{'} \ln x_i t_i$, where $\phi^{'}$ is a vector of parameters. This would, however, introduce a theoretical inconsistency between the functional forms of the non-spatial translog and the local spatial translog. With the spatial autoregressive translog specification, however, technological change need not be Hicks-neutral. This is evident because the spatial autoregressive translog production function for panel data which we estimate is:

$$\ln y_{it} = \kappa + \psi_t + \alpha_i + TL\left(x_{it}, t\right) + z_{it}\vartheta + \lambda \sum_{j=1}^{N} w_{ij} \ln y_{jt} + \varepsilon_{it}. \tag{2}$$

We do not augment Eq. 2 with a spatial error term because we use maximum likelihood (ML) and when ML is used to estimate such a model where the spatial error and spatial autoregressive terms are computed using the same spatial weights matrix, identification of the spatial parameters tends to be weak (Pfaffermayr, 2009, page 67). For the same reason we do not augment Eq. 1 with a spatial error term to obtain the Manski model (Elhorst, 2010, page 14).[6]

We use ML rather than GMM or Bayesian MCMC to estimate the spatial models to maintain commonality with the classic Battese and Coelli (1988; 1992; 1995) stochastic frontier models. This is because a natural area for further work is to develop spatial autoregressive and spatial Durbin stochastic frontier models. We ensure that λ lies in its parameter space, account for the endogeneity of the spatial autoregressive variable and the fact that ε_t is not observed by including the scaled logged determinant of the Jacobian of the transformation from ε_t to $\ln y_t$ in the log-likelihood functions associated with Eqs. 1 and 2 (i.e. include $T \ln |I - \lambda W|$ in the log-likelihood functions, where I is the $(N \times N)$ identity matrix). Moreover, to circumvent the incidental parameter problem associated with the fixed effects Eqs. 1 and 2 are estimated by demeaning in the space dimension which eliminates the fixed effects and the intercept from the fitted models. For details

[6] With regards to Eqs. 1 and 2 we make the following standard assumptions. (i) The parameter space of λ is taken to be $(1/r_{\min},\ 1)$, where $r_{\min}$ denotes the most negative real characteristic root of W and since we use a row-normalised W, 1 is the largest real characteristic root of W. (ii) We assume that W and $(I - \lambda W)$ are bounded uniformly in absolute value. As a result of this assumption the spatial correlation has 'fading' memory (e.g. Kelejian and Prucha, 1998).

on ML estimation of Eq. 2 by demeaning in space see Elhorst (2009), where Eq. 1 is estimated in similar fashion. Lee and Yu (2010) have since shown that demeaning in space to estimate a spatial model with fixed effects which contains the spatial autoregressive variable results in a biased estimate of σ^2 if N is large and T is fixed, which we denote σ_B^2, where the bias is of the type identified in Neyman and Scott (1948). Following Lee and Yu (2010) and Elhorst (2012) we correct for this bias by replacing σ_B^2 with the bias corrected estimate of σ^2, $\sigma_{BC}^2 = T\sigma^2/(T-1)$, which changes the standard errors and t values.[7]

LeSage and Pace (2009) demonstrate that for models such as Eqs 1 and 2 the coefficients on the explanatory variables cannot be interpreted as elasticities because the marginal effect of an explanatory variable is a function of the spatial autoregressive variable. They therefore suggest the following approach to calculate direct, indirect and total marginal effects. We can rewrite Eqs. 1 and 2 as follows, respectively, where the subscript i's are dropped to denote successive stacking of cross sections.

$$\begin{aligned}\ln y_t \;=\;& (I-\lambda W)^{-1}\kappa\iota + (I-\lambda W)^{-1}\psi_t\iota + (I-\lambda W)^{-1}\alpha + \\ & (I-\lambda W)^{-1}\left(\rho_1 t + \rho_2 t^2\right) + (I-\lambda W)^{-1}\left(\Gamma_t\beta + W\Gamma_t\eta\right) + \\ & (I-\lambda W)^{-1}\left(Z_t\vartheta + WZ_t\upsilon\right) + (I-\lambda W)^{-1}\varepsilon_t, \end{aligned} \tag{3}$$

$$\begin{aligned}\ln y_t \;=\;& (I-\lambda W)^{-1}\kappa\iota + (I-\lambda W)^{-1}\psi_t\iota + (I-\lambda W)^{-1}\alpha + \\ & (I-\lambda W)^{-1}\Omega_t\zeta + (I-\lambda W)^{-1}Z_t\vartheta + (I-\lambda W)^{-1}\varepsilon_t, \end{aligned} \tag{4}$$

where ι is an $(N \times 1)$ vector of ones; α is an $(N \times 1)$ vector of fixed effects; Γ_t and Ω_t are $(N \times L)$ and $(N \times K)$ matrices of stacked observations for $TL\,(x_t)$ and $TL\,(x_t, t)$, where $L < K$; β and ζ are vectors of translog parameters for $TL\,(x_t)$ and $TL\,(x_t, t)$; η is a vector of local spatial translog parameters for $W \times TL\,(x_t)$; and Z_t is a matrix of stacked observations for z_{it}.

We set out the approach to calculate the direct, indirect and total marginal effects for the lth component of the translog function in Eq. 3. Using mean adjusted data all the fitted parameters from a local spatial translog function (i.e. Eq. 3 without the spatial autoregressive variable) are elasticities at the sample mean because at the sample mean

[7] Lee and Yu (2010) also show that estimating such spatial models when N and T are both large by demeaning in space and time leads to biased estimates of all parameters, which they propose a correction for. We do not, however, eliminate the time period effects by demeaning in time and regard them as a finite number of additional regressors. This is because in the application we regard T as fixed relative to the size of N so the time period dummies are not a big consumer of degrees of freedom, and if we demeaned in time we would eliminate t and t^2 from Eq. 1 and Eq. 2.

the own and local spatial quadratic and interaction terms are zero. Extending this to Eq. 3, the fitted β and η parameters can therefore be used to directly calculate direct, indirect and total elasticities for the lth component of the translog function. The matrix of direct and indirect elasticities for each unit for the lth component of the translog function is:

$$(I-\lambda W)^{-1}\begin{bmatrix} \beta_l & w_{12}\eta_l & . & w_{1N}\eta_l \\ w_{21}\eta_l & \beta_l & . & w_{2N}\eta_l \\ . & . & . & . \\ w_{N1}\eta_l & w_{N2}\eta_l & . & \beta_l \end{bmatrix}. \tag{5}$$

Since Eq. 5 yields different direct and indirect elasticities for each unit to facilitate interpretation LeSage and Pace (2009) suggest reporting a mean direct elasticity (average of the diagonal elements of Eq. 5) and a mean indirect elasticity (average row sum of the non-diagonal elements of Eq. 5 in the empirical section of this paper). The mean total elasticity is the sum of the mean direct and mean indirect elasticities.

The direct, indirect and total marginal effects for variables in Eq. 1 for which there is no local spatial variable and the corresponding marginal effects for all variables in Eq. 2 are calculated using Eq. 5 but with the off-diagonal elements in the matrix on the right-hand side of Eq. 5 set equal to zero by construction. Consequently, the ratio of the indirect and direct marginal effects is the same for all the variables in the model for which there is no local spatial variable. This is widely considered to be unrealistic and is a key reason why the spatial Durbin model is favoured in the literature.

To compute t statistics for the mean direct, mean indirect and mean total elasticities, LeSage and Pace (2009) propose Bayesian MCMC simulation of the distributions of the elasticities. $1,000$ parameter combinations are drawn from the variance matrix, where each combination consists of random values from a normal distribution with mean zero and standard deviation one. Mean direct, mean indirect and mean total elasticities are calculated for each parameter combination. The mean direct, mean indirect and mean total elasticities which we report are the averages over the $1,000$ draws. Following Elhorst (2012) the associated t statistics are obtained by dividing the reported mean direct, mean indirect and mean total elasticities by the standard deviation across the corresponding $1,000$ mean elasticities.

Furthermore, for any of the 15 spatial weights matrices with a cut-off, we expand the spatial multiplier matrix $(I-\lambda W)^{-1}$ (see Eq. 6), substitute the expansion into Eq. 5 and conduct the above Bayesian MCMC experiment up to a pre-specified order, R (indexed $r=0,1,2,...R$). This yields partitioned mean direct, mean indirect and mean total elasticities across space (i.e. mean no neighbour effects (I), mean first order neighbour effects (λW), mean second order neighbor effects ($\lambda^2 W^2$), etc.).[8] Following Autant-

[8]Partitioning is only possible with a sparsely specified W (e.g. a contiguity matrix) because such matrices give rise to higher order neighbour effects (i.e. second order neighbour effects, third order

Bernard and LeSage (2009), rather than t statistics we report confidence intervals for the partitioned elasticities.[9]

$$(I - \lambda W)^{-1} = I + \lambda W + \lambda^2 W^2 + \lambda^3 W^3 + ... \tag{6}$$

The usual own returns to scale from a non-spatial model and direct returns to scale from the spatial models measure the percentage change in the ith unit's output due to a one percent increase in the ith unit's inputs. Unlike own returns to scale, however, direct returns to scale also include feedback effects i.e. where the ith unit's inputs change which via the spatial multiplier matrix affects the output of first order neighbours, second order neighbours, etc., where some of this effect on neighbours' outputs rebounds and affects the output of the ith unit. Indirect returns to scale refers to the percentage change in the ith unit's output due to a one percent increase in the inputs of all the other units in the sample. Total returns to scale is the sum of direct and indirect returns to scale and is the percentage change in the ith unit's output due to a one percent increase in the inputs of all N units. Since in Eqs. 1 and 2 x is a $(1 \times M)$ vector of inputs indexed $m = 1, 2, ...M$, direct, indirect and total returns to scale (RTS_i^{Dir}, RTS_i^{Ind} and RTS_i^{Tot}) at the sample mean can be calculated as follows.

$$\sum_{m=1}^{M} ex_{m,i}^{Dir} + \sum_{m=1}^{M} ex_{m,i}^{Ind} = \sum_{m=1}^{M} ex_{m,i}^{Tot}, \tag{7}$$

where $ex_{m,i}^{Dir}$, $ex_{m,i}^{Ind}$ and $ex_{m,i}^{Tot}$ are direct, indirect and total elasticities for the mth input, and $RTS_i^{Dir} = \sum_{m=1}^{M} ex_{m,i}^{Dir}$, $RTS_i^{Ind} = \sum_{m=1}^{M} ex_{m,i}^{Ind}$ and $RTS_i^{Tot} = \sum_{m=1}^{M} ex_{m,i}^{Tot}$.

We observe decreasing direct and indirect returns to scale if $RTS_i^{Dir} < 1$ and $RTS_i^{Ind} < 1$, constant direct and indirect returns to scale if $RTS_i^{Dir} = 1$ and $RTS_i^{Ind} = 1$, and increasing direct and indirect returns to scale if $RTS_i^{Dir} > 1$ and $RTS_i^{Ind} > 1$. Since RTS_i^{Tot} is the sum of RTS_i^{Dir} and RTS_i^{Ind}, we observe decreasing total returns if $RTS_i^{Tot} < 2$, constant total returns if $RTS_i^{Tot} = 2$ and increasing total returns if $RTS_i^{Tot} > 2$. We test the null of constant direct, indirect and total returns for the sample average country using one-sided t tests. It should also be noted that the partitioned elasticities cannot be used to calculate partitioned returns to scale. This is because the partitioned elasticities relate to unpartitioned output.

neighbour effects, etc.). With a densely specified W, however, such as an inverse distance matrix with no cut-off (i.e. inverse distance between all units in the sample) there are no higher order neighbour effects. This is because all units are neighbours of one another so there are only first order neighbour effects.

[9] In the published version of the Autant-Bernard and LeSage (2009) working paper (see Autant-Bernard and LeSage, 2011) confidence intervals are reported for the direct, indirect and total marginal effects but not the partitioned elasticities, which is presumably because of space constraints in the published version.

3 Direct, Indirect and Total Curvature and Monotonicity

With reference to Eqs. 1 and 2 the direct, indirect and total curvature and monotonicity propositions are as follows.[10].

1. Direct Concavity in x_i Proposition: Any linear combination of two input vectors for the ith unit, x_i^A and x_i^B, will produce direct output for the ith unit, $y_i^{AB^{Dir}}$, that is no less than a linear combination of $y_i^{A^{Dir}} = f(x_i^A)$ and $y_i^{B^{Dir}} = f(x_i^B)$:

$$f\left(\tau x_i^A + (1-\tau)\, x_i^B\right) \geq \tau f\left(x_i^A\right) + (1-\tau)\, f\left(x_i^B\right),$$

for all $0 \leq \tau \leq 1$.

2. Indirect Concavity in $\sum_{j=1}^N x_j$ Proposition: Any linear combination of the sums of two input vectors across all the jth units, $\sum_{j=1}^N x_j^A$ and $\sum_{j=1}^N x_j^B$, will produce indirect output for the ith unit, $y_i^{AB^{Ind}}$, that is no less than a linear combination of $y_i^{A^{Ind}} = f(\sum_{j=1}^N x_j^A)$ and $y_i^{B^{Ind}} = f(\sum_{j=1}^N x_j^B)$:

$$f\left(\xi \sum_{j=1}^N x_j^A + (1-\xi) \sum_{j=1}^N x_j^B\right) \geq \xi f\left(\sum_{j=1}^N x_j^A\right) + (1-\xi)\, f\left(\sum_{j=1}^N x_j^B\right),$$

for all $0 \leq \xi \leq 1$.

3. Total Concavity in $\sum_{i=1}^N x_i$ Proposition: Any linear combination of the sums of two input vectors across all N units, $\sum_{i=1}^N x_i^A$ and $\sum_{i=1}^N x_i^B$, will produce total output for the ith unit, $y_i^{AB^{Tot}}$, that is no less than a linear combination of $y_i^{A^{Tot}} = f(\sum_{i=1}^N x_i^A)$ and $y_i^{B^{Tot}} = f(\sum_{i=1}^N x_i^B)$:

$$f\left(\omega \sum_{i=1}^N x_i^A + (1-\omega) \sum_{i=1}^N x_i^B\right) \geq \omega f\left(\sum_{i=1}^N x_i^A\right) + (1-\omega)\, f\left(\sum_{i=1}^N x_i^B\right),$$

for all $0 \leq \omega \leq 1$.

4. Direct Monotonicity Proposition: y_i is monotonically increasing in the mth input of the ith unit, $x_{m,i}$, if $\partial y_i / \partial x_{m,i} \equiv ex_{m,i}^{Dir} \geq 0$.

5. Indirect Monotonicity Proposition: y_i is monotonically increasing in the sum of the mth inputs of the other jth units, $\sum_{j=1}^N x_{m,j}$, if $\partial y_i / \partial \sum_{j=1}^N x_{m,j} \equiv ex_{m,i}^{Ind} \geq 0$.

[10] Propositions 1 and 4 are properties of non-spatial and direct production functions but they are referred to here as propositions for reasons of uniformity with the indirect and total curvature and monotonicity propositions.

6. Total Monotonicity Proposition: y_i is monotonically increasing in the sum of the mth inputs of all N units, $\sum_{i=1}^{N} x_{m,i}$, if $\partial y_i/\partial \sum_{i=1}^{N} x_{m,i} \equiv ex_{m,i}^{Tot} \geq 0$.

Having estimated Eqs. 1 and 2, the direct, indirect and total elasticities gives rise to direct, indirect and total translog production functions, which are assumed to be continuously differentiable. If the above curvature propositions hold for the direct, indirect and total translog functions this has important economic implications because it would indicate diminishing marginal productivity of own inputs, neighbours' inputs, and own and neighbours' inputs combined. Using the direct, indirect and total elasticities from Eq. 1, a diagnostic check to see if the curvature and monotonicity propositions hold involves recognising that $\ln y_i$ can be expressed as follows. Equivalently $\ln y_i$ can be expressed in terms of the direct and indirect elasticities from Eq. 2.

$$\ln y_i = f\left(t_i^{Dir}, TL\left(x_i\right)^{Dir}, z_i^{Dir}, t_i^{Ind}, TL(x_i)^{Ind}, z_i^{Ind}\right) = f\left(t_i^{Tot}, TL\left(x_i\right)^{Tot}, z_i^{Tot}\right), \quad (8)$$

where:

$TL\left(x_i\right)^{Dir} = \gamma^{Dir'} \ln x_i + \frac{1}{2} \ln x_i' \Theta^{Dir} \ln x_i$;
$TL\left(x_i\right)^{Ind} = \gamma^{Ind'} \sum_{j=1}^{N} \ln x_j + \frac{1}{2} \sum_{j=1}^{N} \ln x_j' \Theta^{Ind} \sum_{j=1}^{N} \ln x_j$;
$TL\left(x_i\right)^{Tot} = \gamma^{Tot'} \sum_{i=1}^{N} \ln x_i + \frac{1}{2} \sum_{i=1}^{N} \ln x_i' \Theta^{Tot} \sum_{i=1}^{N} \ln x_i$.

The continuity property of $TL\left(x\right)^{Dir}$, $TL\left(x\right)^{Ind}$ and $TL\left(x\right)^{Tot}$ requires symmetry restrictions on the elements of the matrices Θ^{Dir}, Θ^{Ind} and Θ^{Tot}. By applying the arguments of Diewert and Wales (1987) the curvature of the direct, indirect and total translog production functions can be expressed in terms of the direct, indirect and total Hessian matrices at the sample mean or outside the sample mean, H^{Dir}, H^{Ind} and H^{Tot}. Using the indirect case to illustrate the form of the Hessians:

$$H^{Ind} = \Theta^{Ind} - \widehat{ex^{Ind}} + ex^{Ind} ex^{Ind'}, \quad (9)$$

where $\widehat{ex^{Ind}}$ is a diagonal matrix with indirect input elasticities on the main diagonal and zeros elsewhere, where at the sample mean $\widehat{ex^{Ind}} = \widehat{\gamma^{Ind}}$ and $ex^{Ind} = \gamma^{Ind}$; Θ^{Ind} is a matrix of second order indirect input elasticities at the sample mean. Concavity of the direct, indirect and total translog production functions at the sample mean or outside the sample mean requires that H^{Dir}, H^{Ind} and H^{Tot} are negative semi-definite. We can check whether the Hessians are negative semi-definite by checking the sign pattern of the principal minors in the Hessian. A Hessian is negative semi-definite if all the odd-numbered principal minors are non-positive and all the even-numbered principal minors are non-negative. The monotonicity propositions for the mth input at the sample mean and outside the sample mean hold if the relevant element is positive from the column vector of input elasticities. We therefore calculate the proportion of the mth

input elasticities over the sample which satisfy the relevant monotonicity proposition.

4 Application to Aggregate Production of European Countries

4.1 Data and the Spatial Weights Matrices

We estimate Eqs. 1 and 2 using balanced panel data for 41 European countries for the period $1990 - 2011$, using sixteen inverse distance specifications of W. All the data was extracted from version 8.0 of the Penn World Table, $PWT8.0$ (Feenstra *et al.*, 2013a), which was the most recent version of the Penn World Table at the time. Recently, Johnson *et al.* (2013) reestimated a number of classic empirical macroeconomic models using different vintages of the Penn World Table. They conclude that the estimation results are not robust across different versions of the Penn World Table. Here, however, we obtain reasonable estimates of the key parameters from the preferred models using data from $PWT8.0$. In addition, $PWT8.0$ is the first version of the Penn World Table to include data on capital stock. Using an earlier version of the Penn World Table or data from the World Bank would therefore involve estimating real capital stock.

Output is output-side real GDP, y (in 2005 million U.S. dollars at 2005 PPPs, $rgdpo$), where $PWT8.0$ notation for the variable is in parentheses. As recommended in the documentation which accompanies $PWT8.0$ when comparing productivity across countries we use $rgdpo$ rather than expenditure-side real GDP ($rgdpe$) or GDP at 2005 national prices ($rgdpna$) (Feenstra *et al.*, 2013b, page 31). x is a (1×2) vector of input levels. The inputs are number of workers, x_1 (emp), and real capital stock at current PPPs, x_2 (in 2005 million U.S. dollars, ck).[11] z is a (1×3) vector of variables. z_1 is net exports of merchandise as a share of GDP (where $z_1 = csh\_x + csh\_m$ because all the observations for csh_m in $PWT8.0$ are negative to signify that imports are a leakage), z_2 is government spending as a share of GDP (csh_g) and z_3 is a dummy variable for EU membership.[12] The descriptive statistics for the continuous variables are presented in Table 1 and are for the raw data.

[Insert Table 1]

[11] Following the documentation which accompanies $PWT8.0$ (Inklaar and Timmer, 2013, page 13) we use ck as our measure of real capital stock rather than real capital stock at 2005 national prices (in 2005 million U.S. dollars, $rkna$).

[12] We examined how the results for z_1 are affected by how we capture the trade effect by replacing net exports as a share of GDP with trade openness (i.e. $z_1 = csh\_x - csh\_m$) in the non-spatial and the partial spatial Durbin models. Irrespective of whether we use net exports as a share of GDP or trade openness we find no evidence to suggest that the own/direct effect of z_1 is significant. It therefore makes little difference how we capture the trade effect. The models which we report and discuss are those where z_1 is net exports as a share of GDP.

All sixteen specifications of W are row-normalised so spillovers are inversely related to the relative great circle distance between countries. The first specification of W is denoted W_{Dist} and is a dense matrix with the inverse distance between each pair of capital cities as weights. As we noted above there is no scope with such a dense matrix to partition the elasticities. To address this issue and to analyse particular spatial relationships which form part of W_{Dist}, the other fifteen specifications of W are inverse distance matrices with cut-offs. Five specifications of W relate to the nearest $3-7$ capital cities ($W_{3Near}, ..., W_{7Near}$). The other ten use inverse distances to the capital cities of the biggest $3-7$ import and export partners as weights ($W_{3Import}, ..., W_{7Import}$ and $W_{3Export}, ..., W_{7Export}$). The biggest $3-7$ import and export partners are based on the average import and export flows in 2000 U.S. dollars over the period $2000-2011$, where the data is from the IMF Direction of Trade Statistics. We do not use the biggest $3-7$ import and export flows over the period $2000-2011$ as spatial weights because these weights would be endogenous.

4.2 Overview of the Fitted Models, Curvature and Monotonicity

In this subsection we provide an overview of the fitted spatial models, the associated marginal effects and the spatial returns to scale to: (i) make a case for the partial spatial Durbin specification and (ii) argue that the key marginal effects and spatial returns to scale from the partial spatial Durbin models are reasonable. We then discuss the concavity and monotonicity results for the partial spatial Durbin models. In the next subsection we discuss in more detail the direct, indirect and total marginal effects from the partial spatial Durbin models, which is followed by a subsection on the spatial returns to scale from this preferred set of models. In Table 2 we present the Within estimate of the non-spatial model, and selected partial spatial Durbin and spatial autoregressive models (W_{Dist}, W_{5Near}, $W_{5Import}$ and $W_{5Export}$, where the time period effects are not reported).[13]

[Insert Table 2]

To test the null hypothesis that the fixed effects in the partial spatial Durbin and spatial autoregressive models are not jointly significant (i.e. $\alpha_i = ... = \alpha_N = \kappa$) we perform a likelihood ratio (LR) tests on each of the fitted models against the corresponding pooled model. The test statistic is chi-squared distributed with degrees of freedom equal to the number of restrictions which must be imposed on the unrestricted model to obtain the restricted model, which in this case is $N-1$. For all the partial spatial Durbin and

[13]The complete set of fitted spatial models is available from the corresponding author upon request. It should also be noted that when estimating the non-spatial Within model using a standard software package a small number of time period effects were automatically dropped for reasons of collinearity. We therefore dropped the same time period effects from our programs to estimate the spatial models.

spatial autoregressive models we reject the null at the 0.1% level, thereby justifying the inclusion of fixed effects in Eqs. 1 and 2.[14]

Although we cannot interpret the parameters from the partial spatial Durbin and the spatial autoregressive models there are a number of statistically significant local spatial variables in all the partial spatial Durbin models. This is evident for the selected partial spatial Durbin models reported in Table 2. The omission of significant local spatial variables from the spatial autoregressive models gives rise to slightly upwardly biased estimates of λ. For all but one of the sixteen specifications of W the estimates of λ from the partial spatial Durbin models are larger than from the corresponding spatial autoregressive model. That said, on average, the estimate of λ from the partial spatial Durbin models is only 0.06 lower. Also, we find that although the direct, indirect and total marginal effects from the spatial autoregressive models for the $\ln x_1 \times t$ and $\ln x_2 \times t$ variables are all significant at the 0.1% level, in all cases these elasticities are small, with the largest observed elasticities being only ± 0.04. In this application therefore partial spatial Durbin models which assume Hicks-neutral technological change seem reasonable.

The non-spatial model yields own labour and capital elasticities at the sample mean of 0.659 and 0.239, both of which are significant at the 0.1% level. The direct labour and capital elasticities from the spatial autoregressive models are of the order $0.557-0.641$ and $0.236-0.283$, respectively, all of which are significant at the 0.1% level. The direct labour and capital elasticities from the partial spatial Durbin models range from $0.608-0.842$ and $0.238-0.332$, respectively, all of which are once again significant at the 0.1% level. For all the fitted models therefore we find that the own/direct labour elasticity is larger than the capital elasticity, which corroborates the findings from key empirical macroeconomic studies (e.g. Ireland, 2004, and Smets and Wouters, 2003).

We also find that the own/direct labour elasticities are frequently within the range for the labour income share of GDP for the EU 15 member states as reported by the EU Commission ($0.54-0.68$ from Table 1 in Arpaia *et al.*, 2009). Interestingly, the models where the direct labour elasticity lies above this range are all partial spatial Durbin models. That said, for all the partial spatial Durbin models we cannot reject constant direct returns to scale, which is consistent with the assumption of constant returns in classic macroeconomic theories (e.g. Ireland, 2004, and Smets and Wouters, 2003) and with evidence from key empirical studies (e.g. Burnside *et al.*, 1995). In contrast, for all but three of the sixteen spatial autoregressive models we reject constant direct returns to scale in favour of decreasing direct returns. Moreover, from the partial spatial Durbin models we obtain more sensible estimates of indirect returns to scale, where for eleven

[14]To further illustrate for the partial spatial Durbin models. The LR test statistic is 1084.83 using W_{Dist} and the test statistics range from $998.75-1138.97$ for models using $W_{3Near}-W_{7Near}$, from $1023.10-1102.12$ for models using $W_{3Import}-W_{7Import}$ and from $1020.83-1107.32$ for models using $W_{3Export}-W_{7Export}$. The lower (upper) limit of these ranges or any other range reported in this application does not relate to W_{3Near} (W_{7Near}), $W_{3Import}$ ($W_{3Import}$) or $W_{3Export}$ ($W_{7Export}$).

of the sixteen partial spatial Durbin models we cannot reject constant indirect returns and for the other five models there is evidence of decreasing indirect returns. From the spatial autoregressive models, however, we sometimes observe implausibly large indirect returns to scale particularly in light of the corresponding direct returns. For example, the $W_{4Import}-$ $W_{7Import}$ spatial autoregressive models yield indirect returns to scale ranging from $1.570 - 2.182$, whereas the direct returns are of the order $0.848 - 0.921$. All things considered, although the fitted spatial autoregressive models are not without merit we have a preference for the partial spatial Durbin models and so the remainder of this empirical analysis is confined to these models.

Turning now to the spatial concavity and spatial monotonicity results. A production function assumes that the ith unit's output is concave in the ith unit's inputs. Here there is the added issues of whether the ith unit's output is concave/convex in the inputs of the other units in the sample and concave/convex in the inputs of all N units. Applying the above diagnostic check of concavity to the non-spatial, direct, indirect and total translog production functions at the sample mean indicates the following. (i) the non-spatial translog production function and seven of the sixteen direct translog production functions are concave. (ii) Fifteen of the sixteen spatial models yield indirect and total translog production functions which are concave.[15] Interestingly, at the sample mean we find that all the direct $W_{3Import}-$ $W_{7Import}$ translog production functions are concave, and the only indirect and total translog production functions which are not concave relate to W_{Dist}. Our findings therefore suggest that even though direct concavity is a theoretical property of the spatial models, and indirect and total concavity are not theoretical properties, we observe more empirical evidence of indirect and total concavity at the sample mean than we do direct concavity. Having observed that all the indirect translog production functions are concave at the sample mean where W is sparse, we can therefore conclude that at this point in the sample there is diminishing marginal productivity of input spillovers from near neighbours, and big import and big export partners.

Applying the above diagnostic check of concavity to the non-spatial, direct, indirect and total translog production functions outside the sample mean is very revealing. We find that the non-spatial translog production function is concave for 91.0% of the sample and the direct translog production functions are concave for $67.0\% - 100\%$ of the sample. The upper limit of this range relates to the direct translog production functions which are concave at the sample mean. The lower limit relates to the direct W_{Dist} translog production function, which is not concave at the sample mean. The implication is that for direct translog production functions which are not concave at the sample mean, we still observe quite a lot of evidence of direct concavity over the sample. On average,

[15] Where there is no evidence of concavity this does not imply convexity. This is because although the principal minors in the relevant Hessian are not negative semi-definite this does not mean that they are positive semi-definite, which is necessary for convexity.

the W_{Near}, W_{Export} and W_{Import} indirect translog production functions are concave for 90.5%, 65.1% and 61.5% of the sample, respectively. We can therefore conclude that for the three types of sparse W, there is much more evidence over the sample of diminishing marginal productivity of input spillovers from near neighbours.

Having observed that, on average, the W_{Export} and W_{Import} indirect translog production functions are not concave for a non-negligible proportion of the sample we summarise these concavity results over the study period in Figures 1 and 2. Specifically, we present the proportions of the sample for which the W_{Export} and W_{Import} indirect and total translog production functions are concave for countries where we do not observe direct, indirect and total concavity over the entire sample.[16] The most striking feature of Figures 1 and 2 is the nature of the relationship between the proportions pertaining to the indirect and total translog functions. The indirect and total proportions for EU countries follow the same path over the sample. This is also the case for non-EU countries. For non-EU countries, however, the total proportion is similar in magnitude to the indirect proportion over the entire study period, whereas for EU countries the indirect proportion is always below the total proportion. We can therefore conclude that, on average, for the countries which feature in Figures 1 and 2, concavity of the indirect function is the principal driver of concavity of the total function.

[Insert Figures 1 and 2]

A production function also assumes that the *ith* unit's output is monotonically increasing in the *ith* unit's inputs. New lines of enquiry which follow from our spatial translog production functions include whether the *ith* unit's output is monotonically increasing/decreasing in the inputs of the other units in the sample and monotonically increasing/decreasing in the inputs of all N units. At the sample mean, the own input elasticities from the non-spatial model, and the direct, indirect and total input elasticities from all the partial spatial Durbin models are positive.[17] This indicates that at the sample mean the *ith* unit's output is monotonically increasing in the *ith* unit's inputs, the inputs of the other units in the sample and the inputs of all N units. Moreover, the own labour and capital elasticities outside the sample mean from the non-spatial model indicate that, on average, a country's output is monotonically increasing in its own labour and capital for 88.0% and 61.0% of the sample, respectively. On average, the direct, indirect and total labour (capital) elasticities from the sixteen spatial models satisfy the direct, indirect and total monotonicity propositions for 95.0% (79.0%), 85.4% (58.6%) and 94.8% (67.6%) of the sample, respectively. We can therefore conclude that, on average, there is a lot more evidence of direct capital monotonicity from the spatial models than there

[16] Figure 1 relates to 30 EU and non-EU countries and Figure 2 to 26 EU and non-EU countries.

[17] We present and analyse the direct, indirect and total input elasticities from the partial spatial Durbin models in detail in the next subsection.

is of capital monotonicity from the non-spatial model. Also, even though indirect and total monotonicity are not properties of the spatial production functions we still observe a substantial amount of evidence of indirect and total labour monotonicity, and quite a lot of evidence of indirect and total capital monotonicity.

4.3 Further Discussion of the Preferred Models

Since λ is assumed to lie in the parameter space $(1/r_{\min}, 1)$ it cannot be interpreted as an elasticity. However, the estimates of λ indicate how the spatial dependence of $\ln y$ is affected by the specification of W. From Table 2 we can see that the estimate of λ from the W_{Dist} model is 0.313. The average estimates of λ across the $W_{3Near} - W_{7Near}$, $W_{3Import} - W_{7Import}$ and $W_{3Export} - W_{7Export}$ models are 0.399, 0.585 and 0.481, respectively. This suggests that, on average, output dependence between all European countries is less than that between near neighbours, big import partners and big export partners. Furthermore, where sparse specifications of W are used, on average, output dependence is highest between big import partners.

Direct, indirect and total marginal effects for selected partial spatial Durbin models (W_{Dist}, W_{5Near}, $W_{5Import}$ and $W_{5Export}$) are presented in Table 3.[18] We also calculate a corresponding set of partitioned elasticities up to order nine. However, in the Appendix for brevity we only report the partitioned elasticities for the factor inputs for orders $r = 0, ..., 6$.[19] We noted above that the direct labour (capital) elasticity from the W_{Dist} model is 0.842 (0.260) and, on average, the direct labour (capital) elasticities from the W_{Near}, W_{Export} and W_{Import} models are 0.785 (0.255), 0.717 (0.317) and 0.637 (0.305), respectively. This highlights how the specification of W affects the estimates of labour and capital productivity for the sample average country. Moreover, we are able to establish from the partitioned elasticities in the Appendix that the principal component of the direct elasticities is the own (W^0) effect as the higher order effects are always small.

[Insert Table 3]

All the indirect input elasticities are positive. They are also all significant at the 5% level or lower with the exception of three indirect capital elasticities ($W_{3Near} - W_{4Near}$ and $W_{3Import}$). This indicates that the evidence of positive labour spillovers between European countries is robust across the sixteen specifications of W. In addition, it follows from the positive set of direct and indirect labour and capital elasticities that all the total elasticities are positive, all of which are significant at the 0.1% level despite, as we noted above, some of the indirect capital elasticities not being significant. The average indirect labour (capital) elasticities from the W_{Near}, W_{Export} and W_{Import} models are

[18] The direct, indirect and total marginal effects for the other twelve partial spatial Durbin models are available from the corresponding author upon request.

[19] All the unreported partitioned elasticities are available from the corresponding author upon request.

0.478 (0.137), 0.545 (0.207) and 0.745 (0.307), respectively. This indicates that where sparse specifications of W are used, on average, the largest labour and capital spillovers are from a country's biggest import partners.

The direct labour elasticity from the W_{Dist} model is 0.273 greater than the indirect labour elasticity and the direct capital elasticity is 0.081 less than the indirect capital elasticity. For other specifications of W the direct and indirect input elasticities are of a different relative order of magnitude. To illustrate, on average, for the W_{Near} and W_{Export} models, the direct capital elasticity is 0.118 and 0.110 larger than the indirect capital elasticity, respectively. Furthermore, on average, the direct labour elasticity from the W_{Import} models is 0.108 below the indirect labour elasticity, whereas the direct and indirect capital elasticities are essentially the same.

Considering now the average input elasticities over the sample. In Figure 3 we present the average input elasticities from the non-spatial model, and the average direct, indirect and total input elasticities from the W_{Dist} model. We present elasticities from the W_{Dist} model because the specification of W reflects the interaction between a European country and all the other countries in Europe and not just, for example, interaction between a country and its biggest six import partners. We conclude the following from Figure 3. Firstly, over the entire sample the non-spatial, direct, indirect and total labour elasticities are greater than the corresponding capital elasticities. Secondly, from 2003 onwards there is a rise (decline) in the non-spatial, direct and total labour (capital) elasticities, where, in general, the rise (decline) tails off towards the end of the sample.

[Insert Figure 3]

In Figure 4 we present the labour and capital elasticities over the sample from the non-spatial and W_{Dist} models for EU and non-EU countries. Over the entire sample, the direct labour elasticity for EU countries is greater than that for non-EU countries. In contrast, the direct capital elasticity for EU countries is always less than that for non-EU countries. In 2004 there was a fall in the direct labour elasticity for EU countries, whereas there was very little change for non-EU countries. This suggests that labour productivity in the 2004 EU accession countries was low relative to their EU peers. Conversely, there was a rise in the direct capital elasticity in 2004 for non-EU countries, which suggests that capital productivity in the 2004 EU accession countries was below the average for the non-EU cohort. There was, however, very little change in 2004 in the direct capital elasticity for EU countries, which suggests that capital productivity in the 2004 EU accession countries was roundabout the EU average. Interestingly, at the end of the study period the direct capital elasticity for EU countries is approximately zero.

[Insert Figure 4]

With the exception of the direct government size elasticity from the W_{7Near} model which is negative and significant at the 5% level, the direct elasticities for the three z variables are not significant. This is an important finding because our non-spatial model suggests that government size has a significant negative effect and EU membership has a significant positive effect (see Table 2). Our empirical findings for the z variables therefore highlight how spatial models can challenge widely accepted relationships from standard non-spatial models such as the robust negative relationship which Folster and Henrekson (2001) observe between government size and economic growth. The results for technological change paint a similar picture. We find from the non-spatial model that the time parameter is positive and significant at the 0.1% level. In contrast, all the direct time parameters from the spatial models are negative and often not significant. The direct time parameters from the $W_{6Export}$ and the $W_{4Import} - W_{7Import}$ models range from $-0.016 - (-0.009)$ and are all significant at the 0.1% level. We do not, however, interpret this as evidence of technological regress. We instead interpret these results in the context of the application and posit that we observe negative direct time effects because our sample contains a large number of Eastern European countries which underwent major reform during the study period.

4.4 Estimates of Direct, Indirect and Total Returns to Scale

Using the labour and capital elasticities at the sample mean from the non-spatial model we compute own returns to scale in the usual way which we denote RTS. Using the direct, indirect and total labour and capital elasticities at the sample mean from the spatial models, we calculate RTS^{Dir}, RTS^{Ind} and RTS^{Tot} from Eq. 7. In Table 4 we present estimates from individual models of RTS^{Dir}, RTS^{Ind} and RTS^{Tot} at the sample mean, where the t statistics for one-sided tests of the null hypothesis of constant direct, indirect and total returns to scale are in parentheses. For the sample average country, the estimate of RTS is 0.898 and the estimates of RTS^{Dir} range from $0.906 - 1.075$. From the non-spatial model we find that returns to scale are constant as they are not significantly less than 1 at the 5% level (t statistic of -1.42). Similarly, it is evident from Table 4 that we cannot reject constant direct returns to scale for all sixteen spatial models.

[Insert Table 4]

The estimates of RTS^{Ind} and RTS^{Tot} from the spatial models are of the order $0.365 - 1.399$ and $1.356 - 2.312$, respectively. It is therefore evident that we observe a wider range of estimates of RTS^{Ind} and RTS^{Tot} than we do RTS^{Dir}. With the exception of the $W_{3Near} - W_{5Near}$, $W_{3Export}$ and $W_{3Import}$ models, we cannot reject constant indirect returns and constant total returns at the 5% level. For the $W_{3Near} - W_{5Near}$, $W_{3Export}$ and $W_{3Import}$ models we reject constant indirect returns and constant total returns at the 5% level in favour of decreasing indirect returns and decreasing total returns. We can therefore conclude that we observe decreasing total returns for the $W_{3Near} - W_{5Near}$, $W_{3Export}$ and $W_{3Import}$ models because the decreasing indirect returns dominate the constant direct returns.

From a policy perspective our findings on indirect returns to scale suggest that each country should import more from its biggest $4-7$ import and export partners at the expense of imports from, among others, its very near neighbours (nearest $3-5$ neighbours). If this results in an increase in the scale of production for a country's biggest $4-7$ import and export partners and a fall in the scale of production for its very near neighbours, a country's output would rise. This is because as we noted above, indirect returns to scale from a country's biggest $4-7$ import and export partners are constant, whereas indirect returns from a country's nearest $3-5$ neighbours are decreasing.

We draw to a close our discussion of returns to scale by analysing the non-spatial and spatial returns over the study period. In Figure 6 we present for EU and non-EU states annual sample average estimates of RTS, and RTS^{Dir} and RTS^{Tot} from the W_{Dist} model. We note that in Figure 6 the difference between RTS^{Dir} and RTS^{Tot} is RTS^{Ind}. The most striking feature of Figure 6 is the fall in the four types of returns to scale (non-spatial, direct, indirect and total) for EU member states at the time of the EU enlargement in 2004.[20] This suggests that the four types of returns to scale for the 2004 EU accession countries are appreciably lower than for other EU states. Furthermore, Figure 6 post 2004 indicates that the EU enlargement had a persistent impact on the four types of returns to scale for EU states.

[Insert Figure 6]

5 Concluding Remarks and Further Work

Having estimated spatial translog production functions for European countries over the period $1990-2001$ using various specifications of the spatial weights matrix, we followed LeSage and Pace (2009) and calculated the direct, indirect and total marginal effects. Using this approach we related the effect of geography to the factor inputs, which enabled us to extend classic characteristics of production, namely returns to scale and diminishing marginal productivity of factor inputs, to the spatial case. Firstly, we proposed direct, indirect and total returns to scale. Secondly, since the direct, indirect and total marginal effects collectively constitute three translog production functions we performed empirical checks to ascertain if the fitted direct, indirect and total functions were concave. Concavity is a theoretical property of a direct production function, which if it is

[20]In contrast, there is an ever so slight increase in the four types of returns to scale for non-EU states in 2004.

satisfied empirically indicates diminishing marginal productivity of a unit's own factor inputs. However, concavity is not a theoretical property of indirect and total production functions but it is revealing to check empirically if the fitted indirect and total functions are concave. This is because if a fitted indirect production function is concave this indicates diminishing marginal productivity of factor input spillovers. In addition, if a fitted total production function is concave this indicates diminishing marginal productivity of own inputs and input spillovers combined.

The two contributions of this paper are particularly appealing because they are not limited to a production function and the translog specification. For all the other technical relationships (cost, revenue, standard and alternative profit, and input and output distance functions) and other possible functional forms (e.g. Cobb-Douglas, Fourier flexible, Generalized McFadden and Generalized Leontief) we can calculate direct, indirect and total returns to scale from the fitted direct, indirect and total functions. We can also conduct empirical checks of these fitted functions for concavity/convexity by looking at the sign pattern of the principal minors in the Hessian matrices, as we have done here. There is thus scope for wider application of direct, indirect and total returns to scale, and the empirical checks of curvature to other technical relationships and functional forms. Moreover, in other applications the direct, indirect and total marginal effects could be used to extend other classic characteristics of production to the spatial case. For example, in multi-output network industries such as the railways and the airline industry it is common to distinguish between returns to traffic density, returns to scale and returns to scope. The networks which firms operate over in such industries are interconnected and often overlap which suggests that it would be appropriate to define and calculate direct, indirect and total measures of returns to traffic density, returns to scale and returns to scope.

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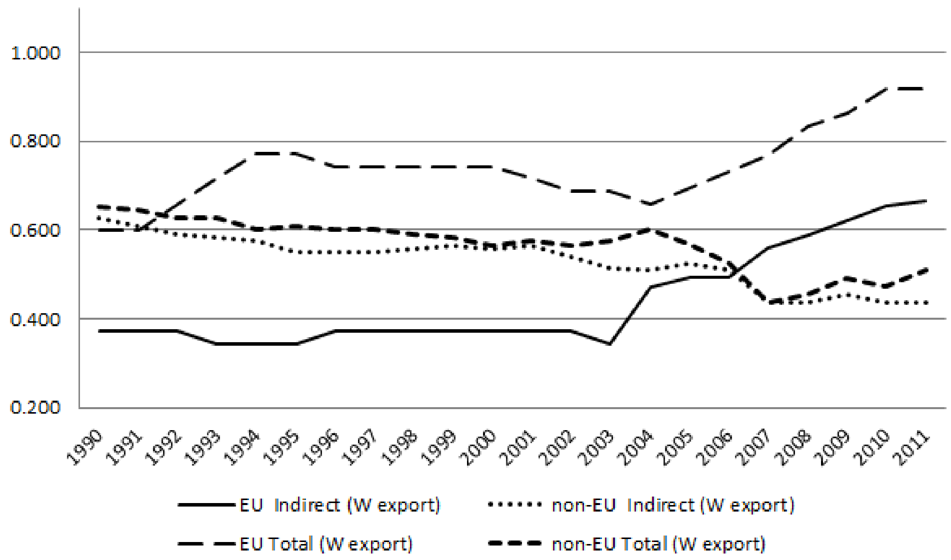

Figure 1: Selected concavity results across the W export models

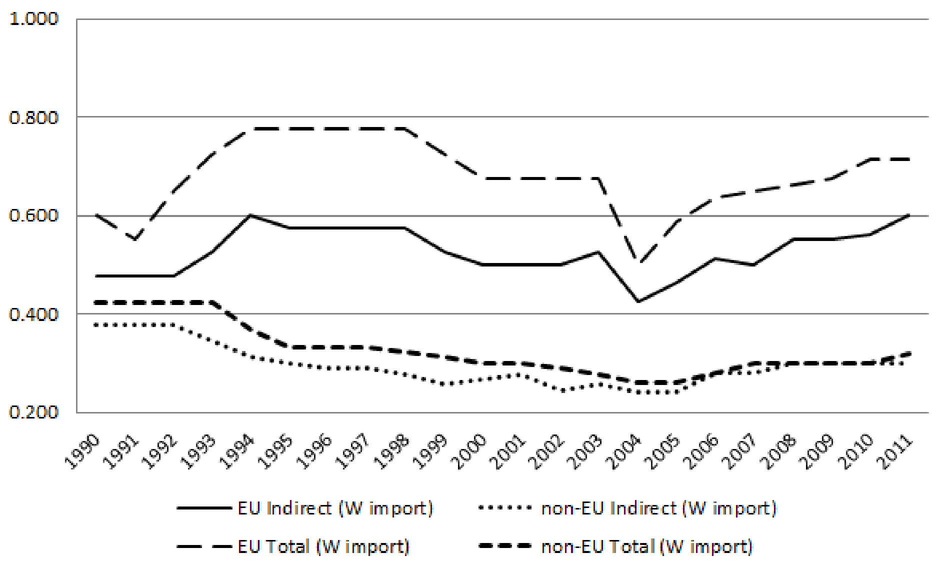

Figure 2: Selected concavity results across the W import models

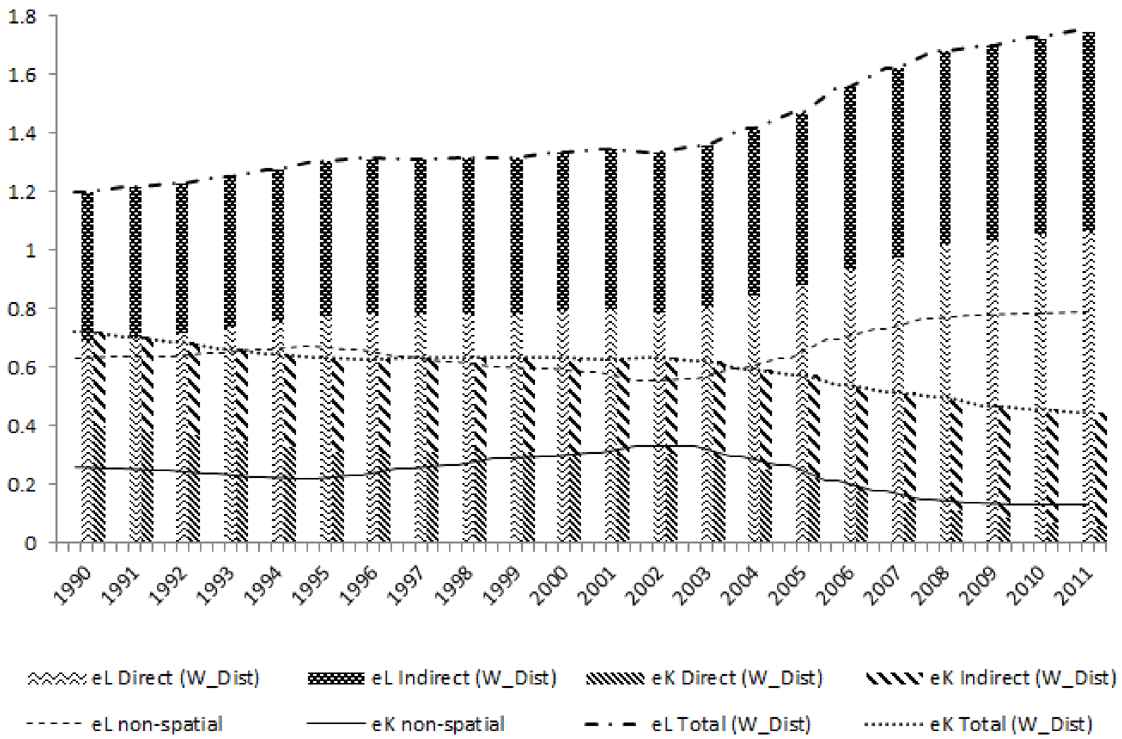

Figure 3: Selected average labour and capital elasticities

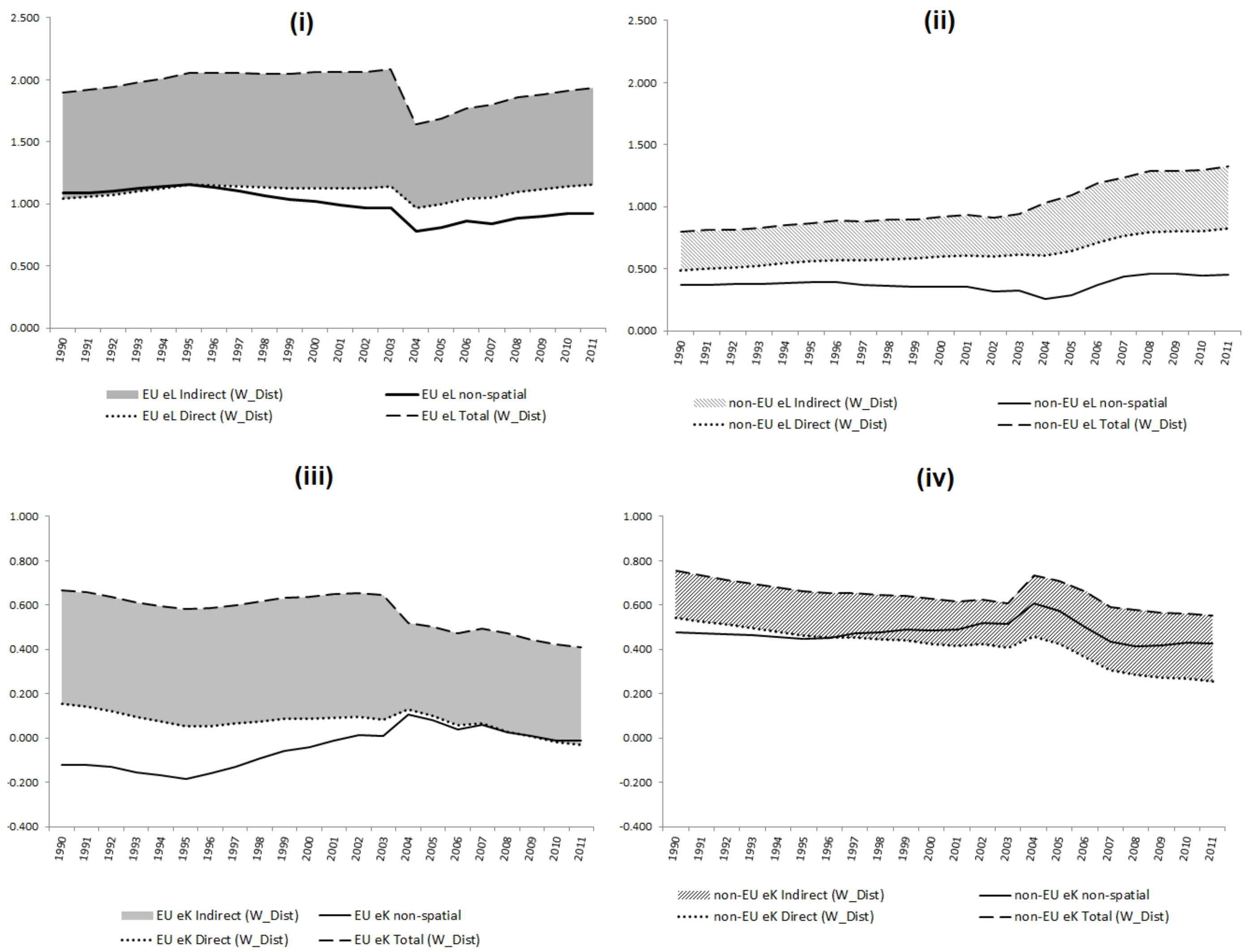

Figure 4: Selected average labour and capital elasticities for EU and non-EU countries

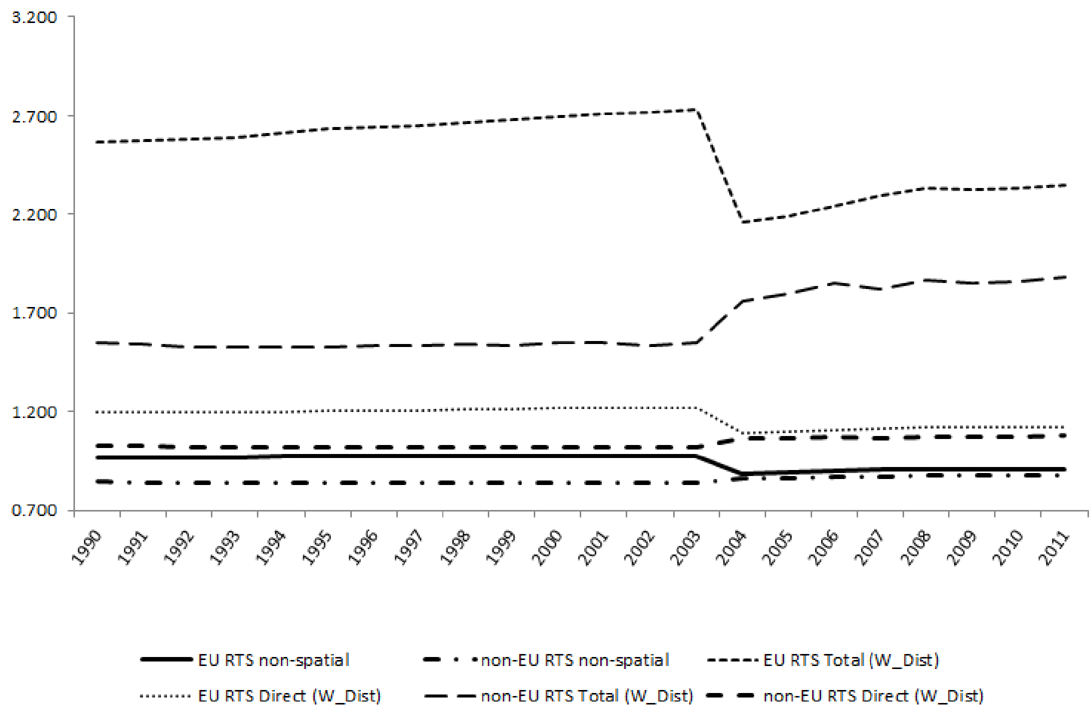

Figure 5: Selected average returns to scale for EU and non-EU countries

Table 1: Summary statistics

	Variable	Mean	St.Dev.	Min	Max
Real GDP (million PPP 2005 U.S. $)	y	$366,380$	$589,103.3$	$4,048.91$	$2,982,019$
Number of workers (millions)	x_1	8.41	13.04	0.13	75.46
Real capital stock (million PPP 2005 U.S. $)	x_2	$1,243,469$	$2,128,382$	$9,205.61$	$10,405,759$
Sum of exports and imports of merchandise as a share of GDP i.e. trade openness	z_1	-0.05	0.13	-0.59	0.67
Government spending as a share of GDP	z_2	0.22	0.08	0.07	0.71

Table 2: Non-spatial model and selected partial spatial Durbin and spatial autoregressive models

	Non-spatial model	PSDM W_{Dist}	SAR W_{Dist}	PSDM W_{5Near}	SAR W_{5Near}	PSDM $W_{5Import}$	SAR $W_{5Import}$	PSDM $W_{5Export}$	SAR $W_{5Export}$
κ	0.184**	–	–	–	–	–	–	–	–
	(3.16)								
$\ln x_1$	0.659***	0.836***	0.628***	0.779***	0.572***	0.574***	0.558***	0.736***	0.598***
	(8.82)	(11.85)	(8.74)	(11.34)	(8.43)	(8.94)	(8.71)	(10.94)	(8.88)
$\ln x_2$	0.239***	0.256***	0.239***	0.248***	0.229***	0.286***	0.264***	0.297***	0.241***
	(6.19)	(6.66)	(6.42)	(6.56)	(6.56)	(7.69)	(7.97)	(7.69)	(6.92)
$(\ln x_1)^2$	−0.337***	−0.188***	−0.340***	−0.245***	−0.339***	−0.238***	−0.293***	−0.245***	−0.308***
	(−10.26)	(−6.02)	(−10.74)	(−8.32)	(−11.41)	(−9.23)	(−10.39)	(−8.26)	(−10.38)
$(\ln x_2)^2$	−0.333	−0.221***	−0.328***	−0.215***	−0.315***	−0.210***	−0.265***	−0.208***	−0.298***
	(−15.43)	(−13.89)	(−15.81)	(−13.77)	(−16.13)	(−15.07)	(−14.30)	(−14.37)	(−15.27)
$\ln x_1 \ln x_2$	0.717***	0.483***	0.708***	0.487***	0.685***	0.451***	0.565***	0.466***	0.635***
	(14.58)	(13.15)	(14.98)	(13.52)	(15.42)	(14.16)	(13.40)	(13.34)	(14.32)
t	0.007***	−0.016	−0.003	0.001	−0.001	−0.012***	−0.006***	−0.005	−0.004*
	(3.43)	(−1.41)	(−1.37)	(0.35)	(−0.67)	(−4.33)	(−3.65)	(−1.74)	(−1.97)
t^2	0.000	−0.003***	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	(−0.51)	(−4.84)	(−1.79)	(−1.09)	(−1.52)	(−1.38)	(−0.51)	(−0.38)	(−0.32)
$\ln x_1 t$	−0.019***	–	−0.018***	–	−0.017***	–	−0.011***	–	−0.015***
	(−7.77)		(−7.82)		(7.76)		(−5.16)		(−6.91)
$\ln x_2 t$	0.018***	–	0.018***	–	0.017***	–	0.011***	–	0.015***
	(7.86)		(7.93)		(8.00)		(5.59)		(7.12)
z_1	0.076	0.044	0.090	0.057	0.117	0.011	0.068	0.110	0.096
	(0.95)	(0.56)	(1.17)	(0.78)	(1.62)	(0.15)	(1.00)	(1.41)	(1.34)
z_2	−0.540***	−0.180	−0.503***	−0.174	−0.506***	−0.182	−0.372***	−0.024	−0.450***
	(−4.50)	(−1.74)	(−4.36)	(−1.61)	(−4.66)	(−1.73)	(−3.62)	(−0.23)	(−4.16)
z_3	0.043*	0.023	0.041	0.010	0.035	0.021	0.020	0.023	0.034
	(1.96)	(1.08)	(1.94)	(0.51)	(1.74)	(1.10)	(1.08)	(1.17)	(1.71)
$W \ln x_1$	–	3.234***	–	0.448***	–	0.268	–	0.161	–
		(6.05)		(3.20)		(1.57)		(0.96)	
$W \ln x_2$	–	0.403	–	−0.018	–	0.042	–	0.062	–
		(1.36)		(−0.21)		(0.51)		(0.79)	
$W(\ln x_1)^2$	–	2.012***	–	0.377***	–	0.380***	–	0.255***	–
		(7.33)		(6.02)		(3.76)		(3.06)	
$W(\ln x_2)^2$	–	0.113	–	0.067*	–	0.144**	–	0.039	–
		(0.97)		(2.10)		(3.17)		(0.84)	
$W \ln x_1 \ln x_2$	–	−0.619*	–	−0.216**	–	−0.298**	–	−0.106	–
		(−2.08)		(−2.94)		(−2.72)		(−0.97)	
$W z_1$	–	−1.342*	–	−0.282	–	0.281	–	0.005	–
		(−2.16)		(−1.63)		(1.30)		(0.02)	
$W z_2$	–	−2.836***	–	−0.417*	–	−1.585***	–	−1.980***	–
		(−3.73)		(−1.98)		(−5.53)		(−6.62)	
$W \ln y$	–	0.313***	0.498***	0.401***	0.428***	0.673***	0.724***	0.471***	0.561***
		(3.61)	(7.64)	(10.52)	(13.83)	(25.25)	(32.31)	(11.85)	(16.95)
Log-likelihood	–	588.505	576.737	614.167	622.23	682.032	660.717	626.802	623.523

PSDM denotes the partial spatial Durbin model and SAR denotes the spatial autoregressive model.
*, **, *** denote statistical significance at the 5%, 1% and 0.1% levels, respectively, where the t-statistics are in parentheses.

Table 3: Marginal effects from selected partial spatial Durbin models

		PSDM W_{Dist}		PSDM W_{5Near}		PSDM $W_{5Import}$		PSDM $W_{5Export}$	
		Marginal Eff	t-stat	Marginal Eff	t-stat	Marginal Eff	t-stat	Marginal Eff	t-stat
$\ln x_1$	*Direct*	0.842***	11.81	0.808***	10.96	0.608***	8.79	0.755***	11.17
	Indirect	0.569**	2.74	0.484***	4.78	0.971**	4.79	0.543***	4.34
	Total	1.411***	6.06	1.292***	8.43	1.579***	6.18	1.298***	7.74
$\ln x_2$	*Direct*	0.260***	6.70	0.256***	6.92	0.298***	7.56	0.303***	7.79
	Indirect	0.341**	3.04	0.133*	2.50	0.425***	3.36	0.174**	2.46
	Total	0.601***	4.80	0.389***	5.05	0.723***	4.67	0.476***	4.98
$(\ln x_1)^2$	*Direct*	-0.187***	-5.96	-0.256***	-8.46	-0.255***	-9.72	-0.254***	-8.44
	Indirect	0.109	1.42	-0.187***	-3.67	-0.606***	-6.04	-0.299***	-4.72
	Total	-0.077	-0.82	-0.442***	-6.19	-0.861***	-7.36	-0.552***	-6.76
$(\ln x_2)^2$	*Direct*	-0.221***	-13.94	-0.226***	-13.72	-0.229***	-15.22	-0.218***	-15.11
	Indirect	0.038	0.47	-0.180***	-3.66	-0.595***	-6.16	-0.290***	-5.07
	Total	-0.183*	-2.15	-0.406***	-7.00	-0.825***	-7.91	-0.508***	-8.05
$\ln x_1 \ln x_2$	*Direct*	0.485***	13.61	0.502***	13.39	0.475***	13.96	0.475***	13.62
	Indirect	0.209	1.72	0.230**	3.17	0.752***	4.99	0.282**	3.16
	Total	0.693***	5.29	0.732***	7.69	1.226***	7.18	0.756***	7.00
t	*Direct*	-0.015	-1.43	0.001	0.37	-0.013***	-4.30	-0.005	-1.76
	Indirect	-0.007	-1.13	0.001	0.36	-0.024***	-3.75	-0.004	-1.64
	Total	-0.023	-1.37	0.002	0.37	-0.037***	-4.02	-0.010	-1.72
t^2	*Direct*	-0.003***	-4.90	0.000	-1.07	0.000	-1.33	0.000	-0.37
	Indirect	-0.001*	-2.12	0.000	-1.03	-0.001	-1.30	0.000	-0.36
	Total	-0.004***	-4.00	0.001	-1.06	-0.001	-1.32	0.000	-0.37
z_1	*Direct*	0.046	0.58	0.056	0.73	0.013	0.16	0.107	1.31
	Indirect	0.001	0.01	-0.006	-0.10	0.003	0.02	0.053	0.62
	Total	0.047	0.36	0.049	0.37	0.016	0.07	0.160	1.01
z_2	*Direct*	-0.180	-1.83	-0.179	-1.63	-0.194	-1.82	-0.021	-0.19
	Indirect	-0.072	-0.97	-0.097	-1.19	-0.320	-1.44	0.019	0.17
	Total	-0.251	-1.62	-0.276	-1.50	-0.514	-1.59	-0.003	-0.01
z_3	*Direct*	0.022	1.02	0.011	0.52	0.022	1.11	0.024	1.20
	Indirect	0.010	0.91	0.007	0.51	0.042	1.09	0.020	1.16
	Total	0.033	1.01	0.018	0.52	0.064	1.10	0.045	1.19

PSDM denotes the partial spatial Durbin model.

*, **, *** denote statistical significance at the 5%, 1% and 0.1% levels, respectively.

Table 4: Spatial returns to scale from the partial spatial Durbin models

Model	PSDM RTS^{Dir}	PSDM RTS^{Ind}	PSDM RTS^{Tot}
W_{Dist}	1.102	0.910	2.012
	(1.58)	(-0.30)	(0.04)
W_{3Near}	0.991	0.365***	1.356***
	(-0.13)	(-6.04)	(-4.21)
W_{4Near}	1.011	0.465***	1.476***
	(0.16)	(-4.51)	(-3.21)
W_{5Near}	1.064	0.617**	1.682*
	(0.90)	(-2.74)	(-1.75)
W_{6Near}	1.063	0.767	1.829
	(0.93)	(-1.51)	(-0.88)
W_{7Near}	1.069	0.862	1.930
	(1.01)	(-0.85)	(-0.35)
$W_{3Import}$	0.978	0.401***	1.378***
	(-0.34)	(-4.48)	(-3.66)
$W_{4Import}$	0.961	0.910	1.871
	(-0.57)	(-0.41)	(-0.50)
$W_{5Import}$	0.906	1.397	2.302
	(-1.40)	(1.45)	(0.96)
$W_{6Import}$	0.914	1.399	2.312
	(-1.25)	(1.45)	(0.99)
$W_{7Import}$	0.949	1.155	2.103
	(-0.73)	(0.64)	(0.36)
$W_{3Export}$	0.991	0.441***	1.432***
	(-0.14)	(-4.25)	(-3.44)
$W_{4Export}$	0.986	0.880	1.867
	(-0.20)	(-0.61)	(-0.56)
$W_{5Export}$	1.057	0.717	1.774
	(0.86)	(-1.54)	(-1.05)
$W_{6Export}$	1.075	0.925	2.000
	(1.06)	(-0.35)	(0.00)
$W_{7Export}$	1.063	0.798	1.861
	(0.91)	(-1.03)	(-0.60)

*, **, *** denote that we reject the null hypothesis of constant returns at the 5%, 1% and 0.1% levels, respectively, where the t-statistics for the tests are in parentheses.

Appendix: Key partitioned elasticities from selected partial spatial Durbin models

		PSDM W_{5Near}			PSDM $W_{5Import}$			PSDM $W_{5Export}$		
Variable	r	Direct	Indirect	Total	Direct	Indirect	Total	Direct	Indirect	Total
$\ln x_1$	W^0	0.779* (0.63, 0.91)	-	0.779* (0.63, 0.91)	0.574* (0.45, 0.71)	-	0.574* (0.45, 0.71)	0.736* (0.60, 0.86)	-	0.736* (0.60, 0.86)
$\ln x_1$	W^1	-	0.312* (0.25, 0.36)	0.312* (0.25, 0.36)	-	0.386* (0.30, 0.48)	0.386* (0.30, 0.48)	-	0.347* (0.28, 0.41)	0.347* (0.28, 0.41)
$\ln x_1$	W^2	0.023* (0.02, 0.03)	0.103* (0.09, 0.12)	0.125* (0.10, 0.15)	0.020* (0.02, 0.03)	0.240* (0.20, 0.32)	0.260* (0.22, 0.35)	0.017* (0.01, 0.02)	0.146* (0.13, 0.18)	0.163* (0.14, 0.20)
$\ln x_1$	W^3	0.004 (0.00, 0.00)	0.047* (0.04, 0.05)	0.050* (0.04, 0.06)	0.005 (0.00, 0.01)	0.170* (0.14, 0.22)	0.175* (0.14, 0.22)	0.002 (0.00, 0.00)	0.075* (0.06, 0.09)	0.077* (0.06, 0.09)
$\ln x_1$	W^4	0.002 (0.00, 0.00)	0.018* (0.01, 0.02)	0.020* (0.02, 0.02)	0.004 (0.00, 0.01)	0.113* (0.09, 0.15)	0.118* (0.09, 0.15)	0.001 (0.00, 0.00)	0.035* (0.03, 0.04)	0.036* (0.03, 0.04)
$\ln x_1$	W^5	0.001 (0.00, 0.00)	0.007* (0.01, 0.01)	0.008* (0.01, 0.01)	0.002 (0.00, 0.00)	0.077* (0.06, 0.10)	0.079* (0.06, 0.10)	0.000 (0.00, 0.00)	0.017* (0.01, 0.02)	0.017* (0.01, 0.02)
$\ln x_1$	W^6	0.000 (0.00, 0.00)	0.003 (0.00, 0.00)	0.003 (0.00, 0.00)	0.001 (0.00, 0.00)	0.052* (0.04, 0.07)	0.053* (0.04, 0.07)	0.000 (0.00, 0.00)	0.008* (0.01, 0.01)	0.008* (0.01, 0.01)
$\ln x_2$	W^0	0.248* (0.17, 0.32)	-	0.248* (0.17, 0.32)	0.286* (0.21, 0.36)	-	0.286* (0.21, 0.36)	0.297* (0.23, 0.37)	-	0.297* (0.23, 0.37)
$\ln x_2$	W^1	-	0.099* (0.07, 0.13)	0.099* (0.07, 0.13)	-	0.192* (0.14, 0.24)	0.192* (0.14, 0.24)	-	0.140* (0.11, 0.17)	0.140* (0.11, 0.17)
$\ln x_2$	W^2	0.007 (0.00, 0.01)	0.033* (0.02,0.04)	0.040* (0.03, 0.05)	0.010* (0.01, 0.01)	0.119* (0.10, 0.16)	0.130* (0.10, 0.18)	0.007* (0.01, 0.01)	0.059* (0.05, 0.08)	0.066* (0.05, 0.09)
$\ln x_2$	W^3	0.001 (0.00, 0.00)	0.015* (0.01,0.02)	0.016* (0.01, 0.02)	0.003 (0.00, 0.00)	0.085* (0.06, 0.11)	0.087* (0.07, 0.11)	0.001 (0.00, 0.00)	0.030* (0.02, 0.04)	0.031* (0.02, 0.04)
$\ln x_2$	W^4	0.001 (0.00, 0.00)	0.006 (0.00,0.01)	0.006 (0.00, 0.01)	0.002 (0.00, 0.00)	0.057* (0.04, 0.07)	0.059* (0.04, 0.08)	0.001 (0.00, 0.00)	0.014* (0.01, 0.02)	0.015* (0.01, 0.02)
$\ln x_2$	W^5	0.000 (0.00, 0.00)	0.002 (0.00,0.00)	0.003 (0.00, 0.00)	0.001 (0.00, 0.00)	0.038* (0.03, 0.05)	0.039* (0.03, 0.05)	0.000 (0.00, 0.00)	0.007* (0.01, 0.01)	0.007* (0.01, 0.01)
$\ln x_2$	W^6	0.000 (0.00, 0.00)	0.001 (0.00, 0.00)	0.001 (0.00, 0.00)	0.001 (0.00, 0.00)	0.026* (0.02, 0.03)	0.027* (0.02, 0.03)	0.000 (0.00, 0.00)	0.003 (0.00, 0.00)	0.003 (0.00, 0.00)

95% confidence intervals are in parentheses.

* denotes significance at the 5% level.