Measuring Efficiency: A Kalman Filter Approach

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Abstract

In the Kalman filter setting, one can model the disturbance term and the inefficiency term of the standard stochastic frontier composed error as unobserved states. This gives significant flexibility to the econometrician when modeling inefficiency. In this study a panel data version of the local level model is used for estimating time-varying efficiencies of firms. Monet Carlo simulation results indicate that whenever the efficiency levels of the firms fluctuate, some of the widely used estimators perform poorly in capturing the efficiencies of the firms. On the other hand, the Kalman filter performs quite well. We apply the Kalman filter to estimate average efficiencies of U.S. airlines during the period 1999-2009 and find that the technical efficiency of these carriers **do not show a tendency to increase. For the first few years of the study period, it seems that the efficiencies of the airlines decreased.**

Keywords: Kalman filter, panel data, airline productivity

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1 Introduction

Stochastic frontier analysis originated with two seminal papers, Meeusen and van den Broeck (1977) and Aigner, Lovell, and Schmidt (1977). Jondrow et al. (1982) provided a way to estimate firm specific technical efficiency. These contributions were framed in a cross sectional data framework. Panel data potentially can give more reliable information about the efficiencies of the firm. Pitt and Lee (1981) and Schmidt and Sickles (1984) applied random effects and fixed effects models to estimate firm specific efficiencies. In these models the efficiencies are assumed to be time-invariant. For long panel data this assumption is might be questionable. The time-invariance assumption was relaxed by Cornwell, Schmidt, and Sickles (1990) (CSS), Kumbhakar (1990), Battese and Coelli (1992) (BC), and Lee and Schmidt (1992). The time-varying inefficiency models were followed by dynamic efficiency models such as Ahn, Good, and Sickles (2000), Desli, Ray, and Kumbhakar (2003), Huang and Chen (2009), and Tsionas (2006). Work on time varying effects models and their use in productivity and efficiency studies have accelerated in the last decade and we view our current contribution as following in this tradition. Many of these advances are summarized in the recent chapter by Sickles, Hao, and Sheng (2013).

In this paper we consider the use of the Kalman (1960) filter by treating both the error term and the inefficiency term as unobserved states. In contrast to the classical Box-Jenkins approach, one also can explicitly model non-stationary stochastic processes in the Kalman filter setting. This gives significant flexibility to the econometrician when specifying the inefficiency portion of the model. We use the Kalman filter estimator (KFE) to model the efficiency component of the stochastic frontier composed error. For this purpose we use a panel data generalization of the local level model. For long panel data, relatively inflexible stochastic frontier models (e.g., BC, CSS, and Kumbhakar (1990)) are more likely to fail to capture potentially complex time-varying patterns of the effects terms. We examine this claim by conducting a series of Monte Carlo simulations. Results of these simulations indicate that some of the widely used estimators can perform poorly in terms of capturing the efficiencies of firms when we have long panel data with fluctuating efficiencies. For example, if the efficiencies of firms are affected by macro factors that tend to have cycles, then it is likely that these relatively inflexible approximators will fail to capture the efficiency patterns. The KFE can be viewed as an alternative to the factor model approach addressed in Kneip, Sickles, and Song (2012) and Ahn, Lee, and Schmidt (2013) and recent generalizations utilizing Bayesian alternatives.

Ueda and Hoshino (2005) appear to have been the first to apply the Kalman filter to the estimation of efficiency in a data envelopment analysis (DEA) framework. Ueda and Hoshino (2005) examine the case where the inputs and outputs are not deterministic. Kutlu (2010**a**), Emvalomatis, Stefanou, and Lansink (2011) and our study appear to be the first to use the Kalman filter to estimate efficiency in the framework of stochastic frontier analysis (SFA).[1] Emvalomatis, et al. (2011) modeled the logarithm of ratio of inefficiency and efficiency by a generalized version of an AR(1) process. Their method, however, does not use the traditional Kalman filter since the state variable is not linearly incorporated in their model, which is a necessary assumption for the traditional version of the Kalman filter. Hence, they use a non-linear version of the Kalman filter. In contrast, we model the effects term as in the local level model and calculate the efficiency scores utilizing the approach adopted by Schmidt and Sickles (1984). Moreover, for our model the traditional Kalman filter method is sufficient for our estimation purposes, although extensions of the Kalman filter, for example, to handle endogenous regressors, recently have been developed and used in a production setting.[2] We apply the KFE to estimate

[1] Our paper is a substantially revised and extended version of Chapter 2 in Levent Kutlu's dissertation, Market Power and Efficiency (2010a). **Recently, independent from us, Peyrache and Rambaldi (2013) proposed a similar Kalman filter model for estimating efficiencies.**

[2] For details see Jin and Jorgenson (2011), Kim (2006), Kim and Kim (2011), Kim and Nelson (2006), and Kutlu and Sickles (2012).

the average efficiencies of the U.S. airlines during the period 1999-2009. **Over our 11 years of study period, the average efficiency of the airlines do not show a tendency to increase. Indeed, for the first few years of the study it seems that the efficiencies of the airlines decreased. As efficiency change and technical (innovation) change are the two main components of productivity growth our empirical findings are broadly consistent with the findings of others (cf, Färe et al., 2007) who report declining service quality as problems with delays and congestion at US major airports accelerated during our sample period.**

In the next section we describe the KFE and propose several ways in which it can be implemented to model productive efficiency. In section 3 we discuss our Monte Carlo simulation results. Section 4 provides the data description and results of an analysis of productivity trends in the US commercial airline industry during the period 1999-2009. Section 5 concludes. **Finally, the Appendix provides additional estimation results for robustness check purposes.**

2 Description of the Kalman Filter Estimator

Consider a panel of I firms observed over n periods. A general stochastic frontier model is given as follows:

$$\begin{aligned} y_{it} &= X_{it}\beta + \mu_{it} + \varepsilon_{it} \qquad (1) \\ \mu_{it} &= T_{\mu}\mu_{i,t-1} + \tau_{it} + e_{1it} \\ \tau_{it} &= T_{\tau}\tau_{i,t-1} + e_{2it} \end{aligned}$$

where $\varepsilon_{it} \sim \mathbf{NID}\left(0, \sigma_{\varepsilon}^{2}\right)$, $e_{it} = \begin{bmatrix} e_{1it} & e_{2it} \end{bmatrix}' \sim \mathbf{NID}\left(0, Q\right)$, and v_{it} are independently distributed error terms. The initial values of the state variables μ_{it} and τ_{it} are assumed to be jointly normally distributed with zero mean and they are independent from ε_{it} and e_{it}. The component μ_{it} is the random heterogeneity specific to i^{th} individual which is interpreted as efficiency. In the spirit of Ahn, et al. (2000) we allow the firm to sluggishly reduce its inefficiency by modeling efficiency as an AR(1) process with trend τ_{it}. We also allow the firm to adjust quickly. Efficiency may be a random walk, for example (cf, Kneip et al., 2012) and thus the model allows for non-stationarity. In our empirical illustration of the KFE that we explore in section 5, **we estimate production efficiency using a restricted translog (RTRANS) production function. Of course the full-translog (FTRANS) functional form can be used and have been used by the authors to estimate airline efficiency but the restricted translog provides us with an empirical vehicle that suits our purpose in this introduction of a new estimator and is statistically supported over the full translog model.**[3]

As a check of the robustness of results based on the restricted translog model we also present estimation results from the full translog model in the Appendix. We calculate the time-varying production frontier intercept common to all producers in period t as $\widehat{\mu}_t = \max_i \widehat{\mu}_{it}$ (Cornwell, et al., 1990). Relative technical efficiency is estimated as $TE_{it} = \exp(-\widehat{u}_{it})$, where $\widehat{u}_{it} = \widehat{\mu}_t - \widehat{\mu}_{it}$. Equation system 1 can be rewritten as:

$$\begin{aligned} y_{it} &= X_{it}\beta + ZB_{it} + \varepsilon_{it}, \qquad \varepsilon_{it} \sim \mathbf{NID}\left(0, \sigma_{\varepsilon}^2\right) \\ B_{it} &= TB_{i,t-1} + e_{it}, \qquad e_{it} \sim \mathbf{NID}\left(0, Q\right) \end{aligned} \tag{2}$$

[3]In the Kalman filter setting it is possible to estimate a cost function with/without input share equations. For the simultaneous equations setting we do not consider a stochastic frontier model because of so called Greene's problem. See Kumbhakar (1997), Kumbhakar and Lovell (2003), and Kutlu (2013).

where

$$B_{it} = \begin{bmatrix} \mu_{it} \\ \tau_{it} \end{bmatrix}, e_{it} = \begin{bmatrix} e_{1it} \\ e_{2it} \end{bmatrix}, T = \begin{bmatrix} T_{\mu} & 1 \\ 0 & T_{\tau} \end{bmatrix}, \text{ and } Z = \begin{bmatrix} 1 & 0 \end{bmatrix}.$$

The Kalman filter equations are given as follows:

$$\begin{aligned}
\eta_{it} &= y_{it} - X_{it}\beta - Zb_{it} \qquad (3)\\
F_{it} &= ZP_{it|t-1}Z' + \sigma_{\varepsilon}^2\\
M_{it} &= P_{it|t-1}Z'\\
b_{it|t} &= b_{it|t-1} + M_{it}F_{it}^{-1}\eta_{it}\\
P_{it|t} &= P_{it|t-1} - M_{it}F_{it}^{-1}M_{it}'\\
b_{it|t-1} &= Tb_{i,t-1|t-1}\\
P_{it|t-1} &= TP_{i,t-1|t-1}T' + Q.
\end{aligned}$$

The corresponding Kalman smoothing equations are:

$$\begin{aligned}
L_{it} &= T - TM_{it}F_{it}^{-1}Z \qquad (4)\\
r_{i,t-1} &= Z'F_{it}^{-1}\eta_{it} + L_{it}'r_{it}\\
N_{i,t-1} &= Z'F_{it}^{-1}Z + L_{it}'N_{it}L_{it}\\
\tilde{b}_{it|t-1} &= b_{it|t-1} + P_{it|t-1}r_{i,t-1}\\
V_{it} &= P_{it|t-1} - P_{it|t-1}N_{i,t-1}P_{it|t-1}
\end{aligned}$$

where $r_{in} = 0$ and $N_{in} = 0$. The log-likelihood is given by:

$$\ln(L) = \sum_{i=1}^{I} L_i = constant - \frac{1}{2}\sum_{i=1}^{I}\sum_{t=d_i+1}^{n}(\ln(F_{it}) + \frac{\eta_{it}^2}{F_{it}}) \qquad (5)$$

where d_i is the number of diffuse states for firm i.

For the initialization of the Kalman filter, one can use the initial values that are implied by stationarity. In the case of non-stationary states, diffuse priors can be used. One practical choice is setting the mean squared error matrix of the initial states to be a constant multiple of the identity matrix. The constant is chosen by the econometrician and should be a large number. Alternatively, one can utilize an exact diffuse initialization.[4] For the sake of simplicity we prefer using the former diffuse initialization method. The traditional Kalman filter estimation may be numerically unstable due to rounding errors which might cause variances to be non-positive definite during the update process. One solution to this issue is using the square-root Kalman filter. Hence, we further implement the square-root Kalman filter.[5]

As we mentioned earlier our model generalizes the traditional random effects model so that the effects term is locally approximated. In order to see this consider the local level model:

$$\begin{aligned} y_{it} &= X_{it}\beta + \mu_{it} + \varepsilon_{it} \qquad (6) \\ \mu_{it} &= \mu_{i,t-1} + e_{it} \end{aligned}$$

where $\varepsilon_{it} \sim \mathbf{NID}(0, H)$ and $e_{it} \sim \mathbf{NID}(0, Q)$. Moreover assume that $Q \rightarrow 0$. Hence, essentially $e_{it} = 0$ and μ_{it} is a deterministic function of initial values, i.e., $\mu_{it} = \mu_{i1}$.

KFE is a random effects-type estimator and is considerably flexible in terms of capturing latent cross-sectional variations that can change over time and which we consider herein unobservable productivity effects. Of course this comes at a price. If the ε_{it} or μ_{it} (effects) terms are correlated with the regressors, then the parameter estimates are

[4] See Durbin and Koopman (2001) for more details about initialization.

[5] See Durbin and Koopman (2001) and Kutlu and Sickles (2012) for details of the square-root Kalman filter.

inconsistent. The KFE can be modified in line with the control function approach used by Kim and Kim (2011) in order to allow for endogeneous regressors that are correlated with the ε_{it} term.[6] Kim (2008) provides a solution to a similar endogeneity problem in the context of markov-switching models when the state variable and regression disturbance are correlated. If the regressors are correlated with the effects term, then we can estimate the first differenced model:

$$\begin{aligned} \Delta y_{it} &= \Delta X_{it}\beta + e_{it} + \Delta\varepsilon_{it} \\ &= \Delta X_{it}\beta + w_{it} \end{aligned} \tag{7}$$

by instrumental variables and standard Kalman filter estimation methods can be applied to the consistent residuals, $y_{it} - X_{it}\hat{\beta}$, in order to obtain the consistent hyperparameter estimates.[7]

3 Monte Carlo Experiments

In this section we implement a set of Monte Carlo simulations to examine the finite sample performance of the KFE. For expositional simplicity we consider a production model. The data generating process is given by:[8]

$$\begin{aligned} y_{it} &= x_{it}\beta + \varepsilon_{it} - \mu_{it}, \quad \varepsilon_{it} \sim \mathbf{NID}\left(0, \sigma_{\varepsilon}^2\right) \\ x_{it} &= Rx_{i,t-1} + \xi_{it}, \quad \xi_{it} \sim \mathbf{NID}\left(0, I_2\right) \end{aligned} \tag{8}$$

[6] **Kutlu (2010b), Karakaplan and Kutlu (2013), and Tran and Tsionas (2012) use similar control function approaches to deal with endogeneity issues in the stochastic frontier context.**

[7] See Harvey (1989) for more details on this type of solutions to the endogeneity problem in **Kalman filter setting**.

[8] When generating regressors we followed Park, Sickles, and Simar (2003, 2007) and Kutlu (2010b).

where $x_{it} = \begin{bmatrix} x_{1it} & x_{2it} \end{bmatrix} \sim \mathbf{NID}\left(0, \left(I_2 - R^2\right)^{-1}\right), \beta = \begin{bmatrix} \beta_1 & \beta_2 \end{bmatrix}' = \begin{bmatrix} 0.5 & 0.5 \end{bmatrix}'$,
$\sigma_\varepsilon^2 = 1$, and

$$R = \begin{bmatrix} 0.4 & 0.05 \\ 0.05 & 0.4 \end{bmatrix}.$$

The generated values for x are shifted around three different means to obtain three balanced groups of firms. We chose $m_1 = (5,5)'$, $m_2 = (7.5, 7.5)'$, and $m_3 = (10, 10)'$ as the group means. We simulate a sample of size $(I, n) = (50, 60)$. Each simulation is carried out $1,000$ times. We consider five different data generating processes for the μ_{it} term:

$$\begin{aligned} DGP\ 1 &: \mu_{it} = \xi_i \\ DGP\ 2 &: \mu_{it} = a_{0i} + a_{1i}(\frac{t}{n}) + a_{2i}(\frac{t}{n})^2 \\ DGP\ 3 &: \mu_{it} = b_{0i} + \sum_{r=1}^{2}\{b_{1ri}\sin(\frac{2rt\pi}{n}) + b_{2ri}\cos(\frac{2rt\pi}{n})\} \\ DGP\ 4 &: \mu_{it} = \eta_t u_i \\ DGP\ 5 &: \mu_{it} = r_{it} \end{aligned} \tag{9}$$

where $\xi_i \sim \mathbf{NID}(0,1)$; $\eta_t = \exp(-h(t-n))$, $h = \frac{0.5}{n}$, and $u_i \sim \mathbf{NID}^+(0,1)$; $a_{li} \sim \mathbf{N}(0,1)$; $b_{lri} \sim \mathbf{NID}(0,1)$; $r_{i,t+1} = r_{it} + v_{it}$ and $r_{i1} \sim \mathbf{NID}(0,1)$; and $v_{it} \sim \mathbf{NID}(0,1)$.

We consider five estimators in our simulations. Each of these estimators correspond to one of the DGPs. The estimators are: Fixed effects (FE) estimator, CSS within estimator (CSSW), Fourier estimator (FOE), Battese-Coelli estimator (BC), and KFE. FE, CSSW, and FOE are described as follows:

$$\hat{\beta} = (X'M_QX)^{-1}X'M_Qy \tag{10}$$

where $M_Q = I - Q(Q'Q)^{-1}Q'$, $Q = \mathrm{diag}(W_i)$, $i = 1, \ldots, I$, and $W_{it} = 1$ for the FE estimator, $W_{it} = [1,\ \frac{t}{n},\ (\frac{t}{n})^2]$ for the CSSW estimator[9], and $W_{it} = [1,\ \sin(\frac{2t\pi}{n}),\ \sin(\frac{4t\pi}{n}),$ $\cos(\frac{2t\pi}{n}),\ \cos(\frac{4t\pi}{n})]$ for the FOE.

Except the BC estimator the technical efficiency is estimated as $TE_{it} = \exp(-\widehat{u}_{it})$, where $\widehat{u}_{it} = \max_i \widehat{\mu}_{it} - \widehat{\mu}_{it}$. The BC estimator assumes that $u_{it} = \eta_t u_i$ where $u_i \sim \mathbf{NID}^+\left(m, \sigma_u^2\right)$ and $\eta_t = \exp(-h(t-n))$. Let $e_{it} = \varepsilon_{it} - \mu_{it}$. For the BC estimator the efficiency is estimated by:

$$\begin{aligned} TE_{it} &= E[\exp(-u_{it})|e_{it}] \\ &= \frac{1-\Phi(\eta_t\sigma^* - \frac{m_i^*}{\sigma^*})}{1-\Phi(-\frac{m_i^*}{\sigma^*})}\exp(-\eta_t m_i^* + \frac{1}{2}\eta_t^2\sigma^{*2}) \end{aligned} \tag{11}$$

where $\eta = (\eta_1, \eta_2, ..., \eta_n)'$, Φ represents the distribution function for the normal random variable and

$$\begin{aligned} m_i^* &= \frac{m\sigma_\varepsilon^2 - \eta' e_i \sigma_u^2}{\sigma_\varepsilon^2 + \eta'\eta\sigma_u^2} \\ \sigma^{*2} &= \frac{\sigma_u^2\sigma_\varepsilon^2}{\sigma_\varepsilon^2 + \eta'\eta\sigma_u^2}. \end{aligned}$$

For the KFE we assume the following model:

$$\begin{aligned} y_{it} &= X_{it}\beta + ZB_{it} + \varepsilon_{it}, \qquad \varepsilon_{it} \sim \mathbf{NID}\left(0, \sigma_\varepsilon^2\right) \\ B_{it} &= B_{i,t-1} + e_{it}, \qquad e_{it} \sim \mathbf{NID}\left(0, Q\right). \end{aligned} \tag{12}$$

Hence, for the KFE the effects term is modelled as a random walk, which is consistent with the local level model of univariate time series. We provide the bias, the variance, the mean squared error (MSE) of the coefficients, the (normalized) MSE of the efficiency estimates as well as the Pearson and Spearman correlations of efficiency estimates with

[9] The original CSSW estimator assumes $W_{it} = [1,\ t,\ t^2]$. However, for the simulations we normalize t by n. This normalization does not affect the results and is done for numerical purposes.

the true efficiency levels. The MSE of the efficiencies are calculated as follows:

$$MSEeff(TE_{0it}, \widehat{TE}_{it}) = \frac{\sum_{i,t}\left(TE_{0it} - \widehat{TE}_{it}\right)^2}{\sum_{i,t} TE_{0it}^2} \tag{13}$$

where TE_{0it} is the true technical efficiency level and $\widehat{TE}_{it}$ is the estimated efficiency level.

The results for the Monte Carlo experiments are given in Table 1-5.

Table 1. Monte Carlo Results for DGP1 (FE)

	FE	*CSSW*	*FOE*	*BC*	*KFE*
MSE	*0.0006*	*0.0007*	*0.0007*	*0.0008*	*0.0006*
$Bias1$	*0.0002*	*0.0002*	*0.0004*	*-0.0046*	*0.0002*
$Bias2$	*0.0000*	*-0.0002*	*-0.0004*	*-0.0049*	*0.0000*
$Var1$	*0.0003*	*0.0003*	*0.0003*	*0.0004*	*0.0003*
$Var2$	*0.0003*	*0.0003*	*0.0004*	*0.0004*	*0.0003*
$MSEeff$	*0.0180*	*0.0528*	*0.0873*	*0.1109*	*0.0720*
$CORP$	*0.9999*	*0.9995*	*0.9991*	*0.9933*	*0.9983*
$CORS$	*1.0000*	*0.9989*	*0.9994*	*0.9987*	*0.9978*

Table 2. Monte Carlo Results for DGP2 (CSSW)

	FE	*CSSW*	*FOE*	*BC*	*KFE*
MSE	0.0007	0.0006	0.0007	0.0009	0.0006
$Bias1$	-0.0009	-0.0008	-0.0010	-0.0077	-0.0008
$Bias2$	-0.0007	-0.0003	-0.0006	-0.0072	-0.0005
$Var1$	0.0004	0.0003	0.0003	0.0004	0.0003
$Var2$	0.0004	0.0003	0.0004	0.0004	0.0003
$MSEeff$	0.0606	0.0413	0.0776	0.1599	0.0955
$CORP$	0.9679	0.9985	0.9905	0.9621	0.9926
$CORS$	0.8882	0.9989	0.9740	0.8623	0.9877

Table 3. Monte Carlo Results for DGP3 (FOE)

	FE	*CSSW*	*FOE*	*BC*	*KFE*
MSE	0.0031	0.0015	0.0007	0.0014	0.0008
$Bias1$	0.0010	0.0006	0.0001	0.0043	0.0006
$Bias2$	0.0004	-0.0002	0.0001	0.0048	-0.0000
$Var1$	0.0016	0.0007	0.0003	0.0007	0.0004
$Var2$	0.0016	0.0007	0.0003	0.0007	0.0004
$MSEeff$	3.2996	0.7278	0.1332	5.6036	0.3621
$CORP$	0.0547	0.3405	0.9705	0.1711	0.8657
$CORS$	0.0152	0.5906	0.9986	0.0599	0.9695

Table 4. Monte Carlo Results for DGP4 (BC)

	FE	*CSSW*	*FOE*	*BC*	*KFE*
MSE	*0.0006*	*0.0007*	*0.0007*	*0.0005*	*0.0006*
$Bias1$	*0.0002*	*0.0002*	*0.0004*	*-0.0050*	*0.0002*
$Bias2$	*-0.0000*	*-0.0002*	*-0.0004*	*-0.0052*	*-0.0001*
$Var1$	*0.0003*	*0.0003*	*0.0003*	*0.0002*	*0.0003*
$Var2$	*0.0003*	*0.0003*	*0.0004*	*0.0002*	*0.0003*
$MSEeff$	*0.0352*	*0.0845*	*0.1336*	*0.0203*	*0.0993*
$CORP$	*0.9985*	*0.9858*	*0.9991*	*0.9890*	*0.9470*
$CORS$	*0.9963*	*0.9826*	*0.9980*	*0.9981*	*0.9421*

Table 5. Monte Carlo Results for DGP5 (KFE)

	FE	*CSSW*	*FOE*	*BC*	*KFE*
MSE	*0.0151*	*0.0041*	*0.0046*	*0.0116*	*0.0014*
$Bias1$	*0.0006*	*-0.0003*	*0.0007*	*0.0351*	*0.0002*
$Bias2$	*0.0003*	*-0.0007*	*0.0002*	*0.0332*	*-0.0013*
$Var1$	*0.0077*	*0.0021*	*0.0023*	*0.0048*	*0.0007*
$Var2$	*0.0074*	*0.0020*	*0.0023*	*0.0045*	*0.0007*
$MSEeff$	*1.0813*	*0.5592*	*0.4246*	*1.3638*	*0.1856*
$CORP$	*0.5032*	*0.7408*	*0.7959*	*0.5288*	*0.9713*
$CORS$	*0.5644*	*0.9634*	*0.9214*	*0.7201*	*0.9975*

For the β estimates, the estimators generally show similar performances. For both the β estimates and the efficiency estimates, whenever there is a high variation in the efficiency term, less flexible estimators, FE and BC, perform worse than others. KFE performs particularly well in terms of correlations between the true efficiency and the estimated efficiency. It is worth noting that all estimators other than the FOE and the KFE performed very poorly for DGP3. Indeed, the FE and BC estimators show almost no correlation between the true efficiency and the estimated efficiency.[10] This is because these estimators are not flexible enough to capture the time-varying pattern of the efficiency. Hence, this simulation study shows that when the efficiencies of the firms fluctuate the performance of non-flexible efficiency estimators can be arbitrarily misleading in capturing the performances of firms.

4 The U.S. Airline Industry 1999-2009

4.1 The Data

We utilize the data from the U.S. airline industry during the period 1999-2009. This is a time period in which the U.S. airlines faced serious financial troubles. The financial losses for the domestic passenger airline operations was more than three times the losses between 1979-1999. Some of the exogenous cost shocks during our sample period are due to increased taxes and jet fuel prices. At the same time fares fell and remained relatively low. In real terms the prices were about 20% lower in 2009 than in 2000. Since 1979 demand grew steadily. However, we observe sharp demand falls in 2001-2002 and 2008-2009 time periods. Due to capital costs and sticky labor prices such unanticipated decreases in demand brought additional complications to an industry which had been experiencing relatively stable and steady demand growth. Another feature of our sample time period is the increase in load factors. Average load factors increased from 71% to 81% between 2000 and 2009 due in part to improved yield management techniques and reduced flight frequency but which also lead to reduced levels of service quality.[11]

The unbalanced data is mainly obtained from the International Civil Aviation Orga-

[10] In some of the simulation runs we observed even negative correlations.
[11] For more information about the financial situations of U.S. airlines see Borenstein (2011).

nization (ICAO). The data set that we use has 35 airlines and 298 observations.[12] We construct our input and output variables following the approaches of Sickles (1985) and Sickles et al. (1986). Our inputs are flight capital (K, quantity of planes), labor (L, quantity of pilots, cabin crew, mechanics, passenger and aircraft handlers, and other labour), fuel (F, quantity of barrels of fuel), and materials (M, quantity index of supplies, outside services, and non-flight equipment's). We focus on value added from capital and labor in our empirical illustration of the KFE by netting out from revenue output (RTK, revenue ton kilometers) the value of the intermediate energy and materials. Thus our technology is rather simple and uses capital and labor to produce value added revenue ton kilometers.

In addition to the above, we include two sets of control variables into our model to account for the heterogeneity of output and the capital input. The first set of control variables is concerned with service characteristics: (i) aircraft stage length (SL) and (ii) load factor (LF). SL is the average length of a route segment, obtained by dividing the miles flown by the number of departures. The shorter (low value) the stage length the shorter the period an airlines' aircraft spends in each flight segment. LF reflects the average occupancy of an airline's aircraft seats, is considered a measure of service quality, and is often used as a proxy for service competition. A lower load factor often implies that the airline assigns a relatively larger number of planes to a particular route and reflects higher service quality by the airline. The second set of control variables is concerned with capital stock characteristics. The first is the average size of the airline's aircraft (SIZE). The larger the size of the aircraft the more services can be provided without a proportionate increase in factors such as flight crew, passenger and aircraft handlers, and landing slots. The second is the percentage of each airline fleet that is a (JET) aircraft to total number of aircraft. JET is considered as a proxy for the aircraft speed. The jet

[12]The full data set has 39 airlines and 321 observations. We droped 1 airline with less than 4 observations and 3 cargo airlines.

aircraft tends to fly around three times as fast as turboprops aircraft and in addition the jet aircraft requires a relatively lower number of flight crew resources. A brief description of the variables is given in Table 6.

Table 6: Description of Variables

Variable	Description	Min	%25 Perc.	%75 Perc.	Max	Mean	Std
Q (y)	ln(Value added RTK)	10.9126	12.6098	15.0266	16.8962	13.9445	1.6004
QL (x_1)	ln(Labor quantity)	3.7351	6.5181	8.5467	10.2650	7.4982	1.5112
QK (x_2)	ln(Capital quantity)	1.9459	3.4410	5.6113	6.7038	4.5095	1.2414
LF (x_3)	Load factor	0.1500	0.5300	0.6210	0.8030	0.5731	0.0990
SL (x_4)	ln(Stage length)	5.5968	6.5164	7.5368	8.5643	7.0730	0.6636
JET (x_5)	Jet engines	0.0000	0.9003	1.0000	1.0000	0.8722	0.2614
SIZE (x_6)	ln(Average plane size)	2.7568	4.0943	5.2244	5.8926	4.7579	0.6020

4.2 Analysis using the KFE

In this section we examine the technical efficiency trends in the U.S. airline industry during the period 1999-2009 using our new KFE and compare our findings to those from the BC and the CSSW estimator. The BC estimator is probably the most widely used of the panel estimators and is a random effects type estimator of efficiency change. The CSSW has somewhat more flexibility and provides a fixed effects treatment. We estimate the value-added production function of the U.S. airlines (revenue ton kilometers less a value weighted average of materials and energy). The production function is specified as linear in logs as:

$$\begin{aligned} y_{it} &= \sum\nolimits_{j=1}^{6} \beta_j x_{jit} + \mu_{it} + \varepsilon_{it} \\ \mu_{it} &= \mu_{i,t-1} + e_{it} \end{aligned} \tag{14}$$

where $\varepsilon_{it} \sim \mathbf{NID}(0, \sigma_\varepsilon^2)$ and $e_{it} \sim \mathbf{NID}(0, \sigma_e^2)$ are independently distributed error terms. The estimates for the production function parameters and average efficiencies for the KFE and the BC and CSSW are given in Table 7 and Figure 1. The overall average efficiencies for the KFE and the BC and CSSW estimators are 0.577, 0.438, and 0.632, respectively. Following Kutlu (2012), any firm with effects term in the upper and bottom $th\%$ range at least at one time period were trimmed.[13] This is a common approach for the regression-based estimators that rely on order statistics. We choose $th = 7.5$ which corresponds to dropping top and bottom three firms.[14] Trimming does not apply to BC since it directly calculates technical efficiencies.

The median of the returns to scale values for the KFE and the BC and CSSW estimators are 0.883, 0.94, and 1.034, respectively. A common finding for the airline industry is that the airlines operate in a constant returns to scale environment. In a single-output production setting, Basu and Fernald (1997) provide a theoretical proof that the value added estimate of returns to scale is smaller (greater) than the corresponding gross output model when there is decreasing (increasing) returns to scale. Hence, there is a magnification effect for returns to scale estimates when a value added production function is used.[15] Therefore, the returns to scale estimate for the KFE might have been driven by this fact. Nevertheless, the constant returns to scale value of 1 lies within one sample standard deviation away from the median value of returns to scale estimates from the KFE. In terms of regularity conditions, KFE outperforms other two estimators. More precisely, while the KFE satisfies

[13]See also Berger (1993) and Berger and Hannan (1995).

[14]We round the numbers up to the closest integer. A common choice is $th = 5$. However, it seems that this was not enough to eliminate the outliers in our case. For the sake of completeness, we provide the efficiency estimates for $th = 5$ in the Appendix.

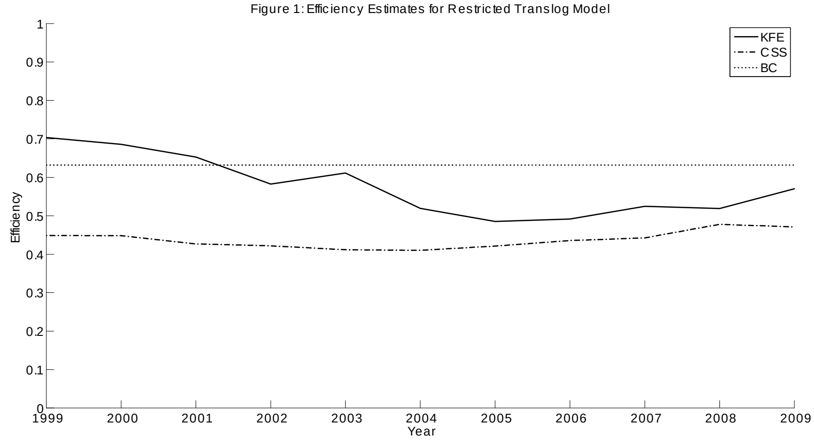
[15]For similar results see also Diewert and Fox (2008).

curvature regularity condition at each time period, the BC and CSSW violate curvature regularity condition at each time period. At the median values of the regressors, all three estimators satisfy monotonicity conditions at each time period. According to KFE estimator, the average efficiency of the U.S. airlines is relatively stable for the second half of the study period. However, there is some evidence in decrease in efficiency for the first half of the study period.

One potential empirical concern would be whether the effects term is correlated with the regressors or not. If the effects term is correlated with the regressors, then the coefficient estimates would be inconsistent. One advantage of the CSSW estimator over the random effects-type estimator is that even when the regressors are correlated with the effects term, the parameter estimates are consistent. Hence, the parameter estimates from the CSSW model might be used to a test the consistency of parameter estimates from the KFE. We test the consistency of parameter estimates from KFE using a Wu-Hausman test and cannot reject the KFE estimates at 5% significance level.

[Table 7 is about here]

Figure 1: Efficiency Estimates for Restricted Translog Model



We also check the robustness of our results by estimating a full version of the translog model. A common problem with the translog production function is that by increasing the number of variables by adding second-order ln **terms to the Cob-Douglas functional form is that the second order terms tend to exhibit considerable multicollinearity. The full translog model estimates are given in the Appendix. For the full translog model, not many of the parameters were significant at** 5% **significance level. We choose our final model specification based on the BIC for the Kalman filter. This criterion is:**

$$BIC = \frac{-2\ln L + \ln(s)(p+d)}{s}$$

where L is the likelihood value, s is the sample size, p is the number of hyperparameters, and d is the number of diffuse priors (Durbin and Koopman, 2001). **The BIC values for the full translog and restricted translog forms are** 1.805 **and** 1.714**, respectively. Based on the BIC and the fact that almost all the parameters of the full translog model are insignificant, we prefer the restricted translog functional form.**

5 Conclusions

In this study we proposed a way to measure technical efficiency via the Kalman filter. Our new Kalman Filter estimator (KFE) provides a local approximation to general time and cross sectionally varying effects terms in a standard panel model. We examine the new estimator in a series of limited Monte Carlo experiments. Our simulation results indicate that while the performance of the KFE is similar to the performances of the other estimators for the coefficient estimates, the KFE outperforms the less flexible estimators in terms of the correlation of the effects with true effects. A result of our simulations is that the widely used BC estimator performed very poorly whenever there is substantial variation in the effects, or for our canonical stochastic frontier efficiency model, the efficiency term. The performance was so poor that sometimes the BC efficiency estimates were negatively correlated with the true efficiency. If the sample data contains events that can cause jumps in the productivity of firms, then the KFE estimator appears able to improve on other standard panel treatments that are less flexible in specifying the temporal variation in the effects. We then used the KFE in order to estimate the average efficiency of the U.S. airlines. **Point estimates for the KFE indicate that average efficiency of the U.S. airlines fell by more than 10% during earlier years of time period. However, the extent of decrease in efficiency was not very robust. Nevertheless, we can confidently claim that the airlines do not improve their efficiency levels for these earlier years. For the second half of our study period, the average efficiency remains relatively constant. This agrees with Färe et al. (2007) who found a decline in service quality since deregulation, yielding in general lower rates of productivity in their sample period 1979I-1996IV.**

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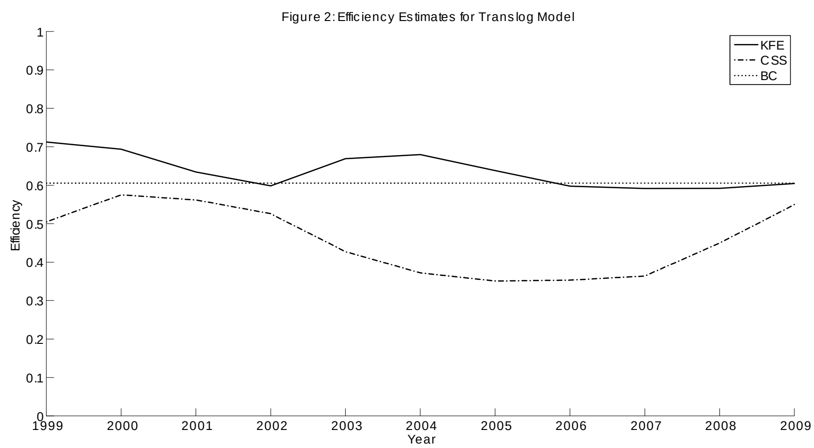
7 Appendix

In this appendix we present additional results based on the full translog model and our truncation scheme when calculating the efficiency estimates for KFE and CSSW estimator. The full translog estimates are given in Table 8. The parameter estimates are generally not significant even at 10% significance level. The median of the returns to scale values for the KFE, CSSW, and BC estimators are 0.8625, 1.1478, and 1.0184, respectively. The corresponding returns to scale estimates from the restricted model were 0.883, 0.94, and 1.034, respectively. Hence, for the KFE and BC estimator the returns to scale estimates are robust to the choice of the functional form. Nevertheless, for both restricted and unrestricted translog production models the constant returns to scale value of 1 lies within one sample standard deviation away from the median value of returns to scale estimates from each of these estimates. All the estimators satisfy the monotonicity conditions at the median values of the regressors at each time period. In contrast to the restricted translog production model where only KFE satisfied the regularity conditions at the median values of the regressors, KFE and CSSW estimator satisfies the curvature conditions at each time period. BC estimator failed to satisfy the regularity conditions at four of the time periods. The estimates for the production function parameters and average efficiencies for the KFE and the BC and CSSW estimators are given in Table 8 and Figure 2. The overall average efficiencies for the KFE, CSSW, and BC are 0.637, 0.458, and 0.605, respectively. These values are not substantially different from their restricted counterparts, i.e.,

0.577, 0.438, and 0.632. The average efficiencies for the full translog model are provided in Figure 2. In line with the restricted translog model, KFE predicts decrease in efficiency in first few years of the study period and relatively stable efficiency levels for the last couple of years.

[Table 8 is about here]



Finally, we present the efficiency estimates when the trimming for KFE and CSSW are done for top-bottom 5% (rather than 7.5%) of the effects term when calculating the efficiencies. The BC estimates remain the same as they are not subject to such trimming. The average efficiency estimates for 5% trimming case are provided in Figure 3 and Figure 4.

Figure 3: Efficiency Estimates for Restricted Translog Model

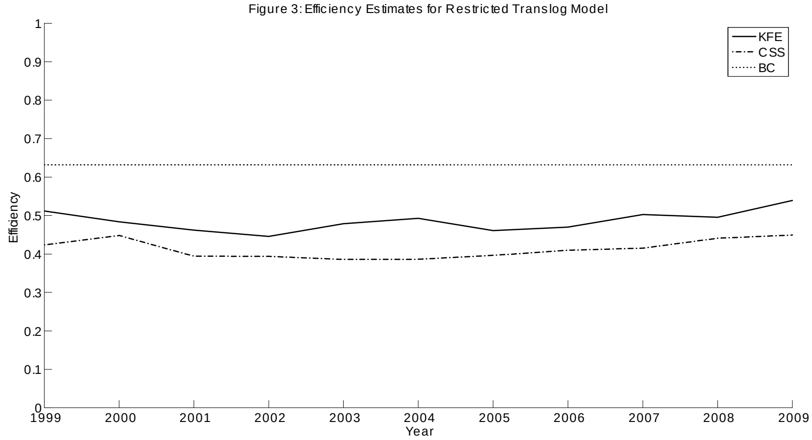


Figure 4: Efficiency Estimates for Translog Model